A DECISION SUPPORT SYSTEM FOR LONG AND SHORT TERM CONTRACTS FOR HYDROELECTRIC COMPANIES IN THE BRAZILIAN ENERGY MARKET

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Dissertação de Mestrado apresentada ao Programa de Pós-graduação em Engenharia Elétrica, COPPE, da Universidade Federal do Rio de Janeiro, como parte dos requisitos necessários à obtenção do título de Mestre em Engenharia Elétrica.

Orientador: Oumar Diene

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A DECISION SUPPORT SYSTEM FOR LONG AND SHORT TERM CONTRACTS FOR HYDROELECTRIC COMPANIES IN THE BRAZILIAN ENERGY MARKET

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Fevereiro/2020

Orientador: Oumar Diene

Programa: Engenharia Elétrica

Apresenta-se, nesta dissertação, um sistema de suporte a decisão destinado a resolver o problema de otimização de carteira de uma companhia hidrelétrica, que considera simultaneamente a alocação de energia no mercado spot, os contratos bilaterais, e a sazonalização da garantia física. O objetivo é maximizar o valor esperado sob restrições de risco, que é feito pelo Conditional Value at Risk (CVaR). Finalmente, um estudo de sensibilidade do *Generation Scaling Factor* (GSF) é feito, uma vez que, este ainda depende das decisões de sazonalização dos outros agentes. O problema é formulado como um programa de otimização linear, e o algoritmo final é implementado utilizando o software MatLab.

Abstract of Dissertation presented to COPPE/UFRJ as a partial fulfillment of the requirements for the degree of Master of Science (M.Sc.)

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In this work, we propose a decision support system to solve the portfolio optimization problem of a hydroelectric company, considering simultaneously the energy allocation in the spot market, the bilateral contracts, and the seasonalization of physical guarantee. The objective is to maximize the expected revenue under a risk constraint, made by hedging the Condition Value at Risk (CVaR). Finally, a sensibility study of the Generation Scaling Factor is made, since it also depends on the decisions (seasonalization) of other agents. The problem is formulated as a linear optimization program, and the final algorithm is implemented using the software MatLab.

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List of Abbreviations

- CVaR = Conditional Value at Risk;
- DSO = Distributing System Operator;
- ERM = Energy Reallocation Mechanism;
- FCA = Free Contractual Ambient;
- GSF = Generation Scaling Factor;
- PG = Physical Guarantee;
- RCA = Regulated Contractual Ambient;
- VaR = Value at Risk;
- SDDP = Stochastic Dual Dynamic Programming;

Chapter 1

Introduction and Bibliographic Review

In the Brazilian electricity market, hydroelectric companies have generally two main kinds of contracts for selling or buying with liquidity in the free market: bilateral (also called forward contract) and short-term contracts (valued at the spot prices), the balance between these contracts is an important decision for the agents, since it has a direct impact on the company's revenue. Furthermore, the total energy from the generation that can be sold is called physical guarantee (PG) [1]. The PG is the amount of energy that can be produced by the company with some confidence level, this amount of energy is calculated by EPE (federal agency of research on energy) and regulated by ANEEL (federal agency of electrical energy regulation). This is an unique feature of the Brazilian electricity market, due to the nature of its central operation dispatch, and dependence on cascade hydroelectric power plants. Another unique feature of the Brazilian energy market, due to its characteristics, is the existence of a mechanism to share the hydrological risk (inflow risks) between the hydroelectric companies, called the Energy Reallocation Mechanism (ERM) [2]. If a hydroelectric company is participating in this mechanism, at the end of each year, it has the right, but not the obligation, to allocate its PG monthly (this process is called seasonalization) during the next year [3]. The seasonalization decision is made under great uncertainty due to price fluctuations and the decision of other companies, that will change the generation scaling factor (GSF), that has a direct influence on the hydroelectric generation companies revenue. The GSF is given by:

$$GSF = \frac{\sum HG}{\sum PG}, \tag{1.1}$$

where $\sum HG$, $\sum PG$ are the total hydraulic generation, and seasonalized PG of all companies participating in the ERM, respectively. As it can be noticed from (1.1), the GSF will be affected by the total ERM generation, which is a decision of the national independent system operator (ONS), and the total physical guarantee allocated each month by the other market players strategy.

The GSF is necessary to calculate the *effective* PG, defined as the actual energy sold by companies participating in the ERM (in contrast to the PG, defined as the energy planned to be sold by a company in a month) and is given by:

$$PG_{eff} = PG \times GSF. \tag{1.2}$$

In practice, the seasonalization is made considering the experience and knowledge of market agents. However, the spot prices and the GSF can be highly volatile, as shown in Fig. 1.1 and Fig. 1.2. Using this methodology the decision is usually sub-optimal, thus the revenue could still be improved. Furthermore, by not considering extreme events, such as high spot prices relative to their expectation, the generation company may be under risk of losing a very large amount of money, which eventually could lead to bankruptcy. In order to avoid this sub-optimal revenue or huge losses, it is possible to formulate this decision problem as an optimization problem with risk constraints.

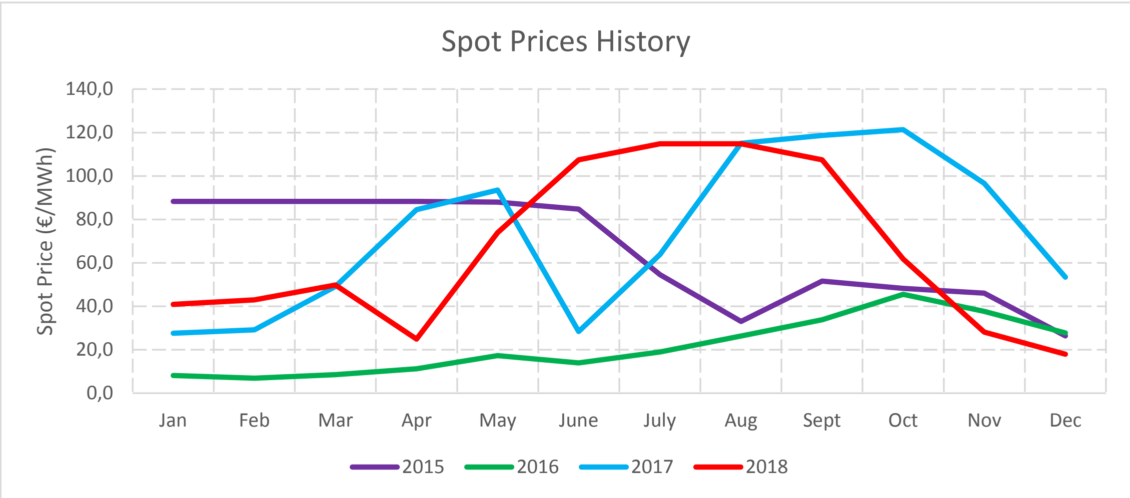


Figure 1.1: Spot Prices history

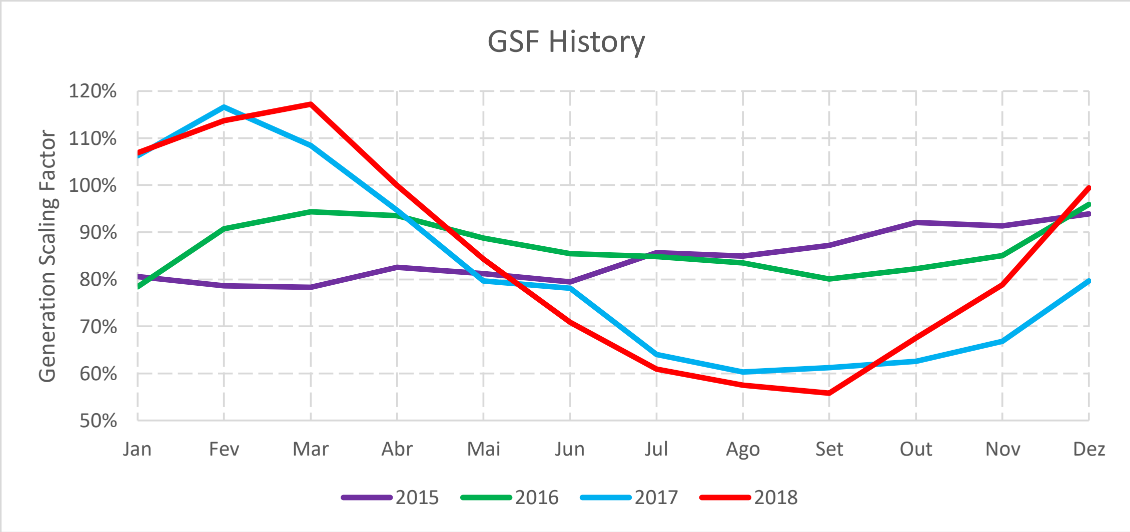


Figure 1.2: GSF history

In the matter of risk analysis, one of the first contributions is the Markowitz mean-variance portfolio theory [4]. It is still used nowadays as a risk measure to the energy market as in [5], where a risk management portfolio of a thermal power plant company is presented. In [5], the risk is defined as the prices differences between electrical nodes/regions, caused by congestion of electric transmission lines, additionally to the spot prices volatility. Since the Markowitz theory, a significant number of risk measures and different risk analysis have been developed. Among them, two of great importance are the Value-at-Risk (VaR) [6], and the Conditional-Value-at-Risk (CVaR) [7]. The risk measure utilized by Markowitz (minimal variance) is not so adequate for the products in the Brazilian energy market. This is mainly due to the fact that all financial contracts have a positive correlation, influenced by the spot prices. Since the risk measures such as the VaR, and CVaR synthesize in one number the effective capital in risk, they are more suitable to the problem. Studies have been made to analyze VaR and CVaR measures, for instance, in [8], a benchmark in the state of art about the analysis methods for risk management in energy markets is proposed. The risk is defined as the spot price volatility, and the risk measures analyzed are the VaR and CVaR. In [9], the methodology proposed takes place from the view point of an energy trading company. The authors compare three risk measures, VaR, CVaR, and medium variance. Finally, it suggests using the three different risk measures that can capture diverse views of the situation to aid the decision maker. In [10], a methodology of multi objective optimization to find the efficient Pareto frontier is developed. The Pareto efficient frontier is a graphical display of the points, that maximize the expected revenue for a given value of risk, the graph displays all the possible points from completely risk averse to risk neutral. The Pareto frontier is utilized to visualize clearly the risk associated with each expected revenue. The motivation of this frontier is to show the risk and return trade-off of every optimal portfolio solution.

There have been studies that try to incorporate the CVaR analysis in the stochastic optimization problem, with the decomposition techniques, that can be for example, the nested decomposition, or the Stochastic Dual Dynamic Programming (SDDP). One of the first works in this area is [11], where the CVaR is presented along with the SDDP method, and the SDDP method is extended to include a risk averse formulation of multistage stochastic programs. In this work, the β CVaR constraint is moved into the objective function, i.e., the authors consider the following equation as the objective function:

$$f_\lambda = (1 - \lambda)E[x] + \lambda CVaR_\beta[x], \tag{1.3}$$

where λ is a tuning parameter based on the preferences of the agent, x are the decision variables, e.g., the amount of buying or selling contracts, and $E[x]$ is the expected revenue based on that decision.

The objective function does not only consist of the expected return, but the risk plays a direct role on the objective function. Additionally, the parameter $\lambda \in [0, 1]$ should be tuned for a compromise between the expected revenue and the risk. The advantage of this formulation over the one considering the CVaR as constraint, is that the latter can make the problem infeasible. However, on the other side, the parameter λ is not very intuitive for the decision maker, since it would be much easier for the agent, to quantify a monetary amount that could be lost, than to quantify an abstract percentage value between expected return, and the CVaR. That is the reason why, in this work, we choose to formulate the CVaR as a direct constraint in the optimization problem. Finally, even if the CVaR constraints turns out the problem infeasible, it is a useful piece of information for the decision agent.

Further development is made in [12] by modifying the use of CVaR in the problem, differing from the concept presented in [11], that made use of the so called polyhedral formula. It replaces the risk measure computation at each stage, by an expectation with an adjusted probability measure. The advantages of this construction (compared to the one presented in [11]), is to avoid the necessity to maintain a VaR state variable at each stage, while the drawback is the expense of having to compute the probability adjustment. However, this formulation is well suited for the decomposition techniques. In [13] this approach is extended to any coherent risk measure, specifically for an application in the Brazilian energy market. This concept was explored in [14] to optimize the portfolio of a company, and to guide the seasonalization process.

While the papers [11]-[14] explored the concepts of stochastic decomposition techniques, and different other approaches to include the CVaR in the objective function, in this work the problem is solved using a different approach. We formulate the problem as a linear optimization program, and solve it for all constraints and scenarios. The reasons to justify this approach are that linear optimization solver can be easily scalable to solve efficiently even problems with millions of constraints and variables, furthermore linear programming do not depends on the construction of approximations, neither in the gap of convergence at each iteration or stage. In the proposed formulation it is proven that even considering many uncertainty scenarios, the algorithm is capable to find an optimal solution in a reasonable time. Furthermore, as already stated, the CVaR is formulated as a direct constraint in order to effectively control the risk, and not just to tune a parameter that is not intuitive to the decision agent.

In many energy market studies the focus is to reduce the risk based on complementarity of hydroelectric with either thermal, wind, or solar power sources, instead of analyzing the financial energy market. Generally the studies are rather interested in the generation curves, from the view point of an investor. In this case, the Markowitz portfolio theory is suitable. In [15], in order to reduce the risks, an hedging mechanism, that uses contracts with thermal power plants, is developed for a hydro generation company. Although the paper analyzes the Brazilian energy market, its reproducibility is not guaranteed since a proprietary model was used. Additionally, the mathematical formulation is not presented. In [16] a portfolio, which takes advantage of the complementarity of a small hydro and biomass generation curves to hedge against the exposition in the market is presented. In this case, the problem is formulated as a linear optimization program, that aims at maximizing the expected revenue of a small hydroelectric company, considering the CVaR as a constraint, but it doesn't take into account the ERM. In [17] a portfolio composed by wind energy's contracts, which is used to complement its hydroelectric generation is presented. However, the formulation is rather incomplete in order to be easily reproducible. In [18], it is shown that the source diversification in [15], [16], and [17] might not always be the best approach, instead, instruments of an energy financial markets may be more efficient to hedge risk than renewable investments. Therefore the instruments to hedge risk considered in this thesis will only be financial instruments.

The portfolio optimization problem from the hydroelectric company's point of view, in the Brazilian market is based on the assumptions that these companies are allowed to buy/sell energy at prefixed prices at the beginning of a period (one month or more), or at the spot price at the end of each month. The total revenue of the company is straightforwardly given by the difference between the total energy sold at each price and the total energy bought at each price. The total energy available to be sold by the company is given by its effective PG plus its total buying forwards contracts. The available energy has to be sold in the market by establishing selling forward contracts at a prefixed price at the beginning of the period or at price at the end of each month. This problem has been solved with respect to maximize the revenue [19], [20], [21], and [22]. For instance, in [19], the author formulates the portfolio problem, in order to optimize the decision between selling its PG in forwards, or waiting to sell its PG at the spot price, and the risk is measured by the CVaR. In another study [22], the authors maximize the revenue from different agents' points of view (generation, commercialization, and consumption), with risk management instruments such as contractual clauses of flexibility, and swaps. On the other hand, this problem has also been solved to allocate optimally the PG [23]. Concerning the seasonalization, in [24], the author formulates the problem of minimizing the risk of a hydroelectric company in the spot market, considering the possibility of PG seasonalization, it defines the risk as the *annual loss* of the PG distribution between the spot market and forwards contracts. In [25], the author uses a stochastic decomposition method (SDDP), to solve the optimization problem of seasonalization of a small hydroelectric plant, along with the decision of selling long term forwards, in the latter there is no risk control, and the objective is only to maximize the expected revenue of the company.

Numerical comparisons were not made with these other studies since the information concerning spot prices, bilateral contracts, or other critical details were not available in order to make a reliable comparison of them with our methodology.

The portfolio problem of a hydroelectric company however, has not yet been solved with both criterion of optimizing the portfolio and seasonalization, while taking into account the CVaR control. In this work, a decision support system is proposed to solve the problem considering simultaneously the energy allocation in the spot market, the bilateral contracts, and the seasonalization of PG. The risk is defined as the spot prices volatility. The objective is to maximize the expected revenue under a risk constraint, made by hedging the Condition Value at Risk (CVaR), a coherent risk measure. Finally, a sensitivity study of the GSF is made, since it also depends on the decisions (seasonalization) of other agents. The problem is formulated as a linear optimization program, and all the mathematical proof, including the formulation of equivalents optimization problems are made. The final algorithm is implemented using the software MatLab.

This thesis is organized as follows: chapter 2 presents the general structure, and all peculiarities of the Brazilian Energy Market that are relevant to a hydroelectric company located there. Chapter 3 provides the necessary mathematical background for understanding risk measures, and optimization tools used to solve the portfolio problem, additionally a simple example is presented in order to illustrate the concepts. Chapter 4 gives a detailed view of the portfolio problem's mathematical formulation. Chapter 5 illustrates, and solves some study cases. Finally, chapter 6 discusses the findings from the previous chapter, and sums up the conclusions.

Chapter 2

Brazilian Energy Market

2.1 Introduction

The aim of this chapter is to give a brief overview of the Brazilian energy market structure and main agents, and to explain in details some key concepts of tools and regulation that the hydroelectric company is subjected to.

2.2 General overview

The Brazilian energy market started its process of deregulation in the late nineties, following three main objectives:

- Solve the fiscal deficit, by selling physical assets;
- Re-establish the investments flow into a investment program;
- Increase the efficiency of the energy companies.

The focus of the restructuring model is the de-verticalization of generation, transmission and distribution companies, the introduction of competition in the generation and commercialization sectors, and the tariff regulation in the transmission and distribution, which are considered natural monopolies [26].

The competition is introduced in this market by establishing the coexistence of two separated markets structures, the Regulated Contractual Ambient (RCA), and the Free Contractual Ambient (FCA). In the RCA, the Distribution System Operators (DSOs) buy energy from auctions, which can be for long or short term, depending on their growth demand projections (four to six years ahead for long term, and one to two years ahead for short term). The DSOs assume the operational risk and are in charge of supplying energy to the final consumers (residential, industrial and commercials consumers). On the other side, for energy consumers with a sufficient demand (over $500kW$), there is the possibility to migrate from the RCA to the FCA, in this case, the energy provider will no longer be the DSOs, but rather the electric generation companies. The consumers can establish bilateral energy contracts, directly with the generators, or with the aid of a commercialization company. The consumers and generators in the free market are exposed to the risks, and specificities of the FCA. A hydroelectric generation company in the FCA will be studied in this work.

2.2.1 Main Agents

Fig. 2.1 describes a general overview of the main energy agents in the Brazilian market.

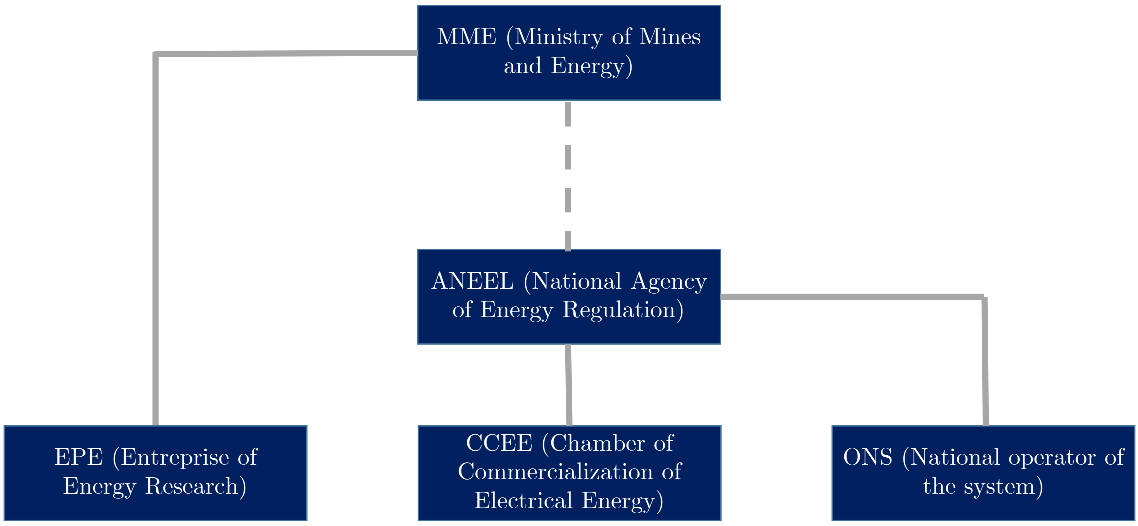


Figure 2.1: General Schema of the Brazilian energy market

The Ministry of Mines and energy (MME) is a government organ responsible for energy policies implementation. Among its assignments, there are directives establishment for energy auctions, definition of concession contracts, acknowledgment of authorization acts, and the PG definition.

The Enterprise of Energy Research (EPE) subordinated to the MME, has the objective to provide studies, research and establish technical support to the energy planning. Among its assignments, the EPE develops energy matrix evolution studies, plans the transmission and generation expansion. Moreover, the EPE plays an important role, by following the MME directives, in order to provide technical studies to establish the PG.

The National Agency of Electrical Energy (ANEEL) is the federal energy regulator, subordinated to the MME. Among its assignments, the organ must regulate and supervise the production, transmission, distribution, and commercialization of electrical energy, following the MME policies and directives.

The National System Operator (ONS, that is also the transmission system operator - TSO) is responsible for the optimal system operation, taking into account the system as a whole, the dispatch decision of each power plant is defined by this central operator.

The Chamber of Commercialization of Electrical Energy (CCEE) has the objective to enable the energy commercialization in the RCA and FCA. Furthermore, it is responsible for accounting all energy contracts, and to monthly audit the quantities of contracted energy versus the effective generation, or consumption by the market agents, and to define the weekly spot price. The CCEE also defines debits and credits of these agents based upon the differences stated above, by the financial liquidation of the operations. In order to valuate the differences it is used the spot price, called Difference Liquidation Price (DLP, or PLD in Portuguese).

2.3 Liquidation

The CCEE has the important assignment, in a weekly basis, of measuring the energy balance of each market agent. All the data from the Energy Reallocation Mechanism (ERM), contracts and measurements are needed to establish the energy balance.

The difference between the verified energy (generation and consumption) versus the contracted energy (selling and buying contracts) results in a deficit or surplus for each agent, that is called the *Balance*. This balance is multiplied by the PLD, and could either be a debit or a credit to the energy agent, in order to the final net energy balance be zero. In this scenario, the CCEE will be the ultimate seller (or ultimate buyer) of energy, which consists in the process called *Liquidation*. The mathematical equation of the balance is given by:

$$Balance = \sum Generation - \sum Consumption + \sum C_{sell} - \sum C_{buy} + ERM \quad (2.1)$$

while the liquidation is given:

$$Liquidation = PLD \times Balance \quad (2.2)$$

This is the main step of determining the short term market liquidation. The detailed commercialization rules may be found in [27].

2.4 Spot Price

The Differences Liquidation Price (DLP) is the spot price equivalent in this market. The definition is rather different from other markets, first of all, the spot prices in the Brazilian market are a result (dual variable) of an optimization problem of the whole integrated electric system optimal operation. The prices are not defined from a competitive bid-based dispatch market, but rather from optimizing the whole integrated system. This optimization is made by the CCEE (Commercialization Chamber of Electrical Energy), which receives the input (demand and inflows projections, transmission conditions, and other agents data) from the ONS. The prices are weekly published, divided in three different hourly bases, and separated for each one of the four sub-markets.

Hence the projection of spot, or forward prices are made by using the same optimization programs utilized by the ONS and CCEE in the system operation, and spot price definition. Although the price projection won't be addressed in this paper, it has been studied in the literature [28, 29].

2.5 Physical Guarantee

The concept of Physical Guarantee (PG) is important to the operation and planning of the energetic system, as well as for the energy commercialization. For the operation and planning, the PG is defined as the energetic benefit that the power plant aggregates to the system. For the commercialization, it is defined as the energy quantity that can be compromised in contracts. This value is defined by the MME, is calculated by the EPE, and can be revised after a five years period. The methodology to calculate the PG is defined in [30]. An example of PG of a hydroelectric plant is depicted in Fig. 2.2.

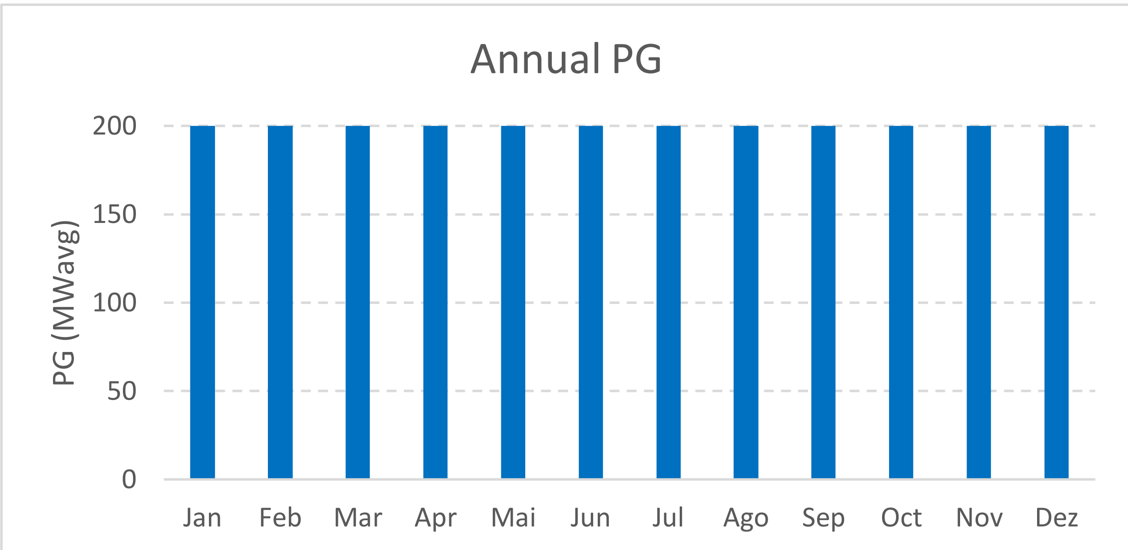


Figure 2.2: Annual PG

2.5.1 Seasonalization

The *Seasonalization* process is the distribution of the annual PG along the months of the year. If the hydroelectric power-plant is participating in ERM (Energy Relocation Mechanism), it has the right to allocate its annual value of PG among the months. It is an additional measure to aid the agent's strategy along the next year, an example of this process is illustrated in Fig. 2.3. However, this process must respect some regulatory limits [1], [3]:

- The annual average sum of the monthly PG seasonalized (PG_m) cannot be higher than the regulated PG (PG_r).
- The monthly PG seasonalized cannot be higher than the hydroelectric power (P_{HE}).
- The hydroelectric company must have 100% contracted energy each month ($\lambda_m \geq 0$).

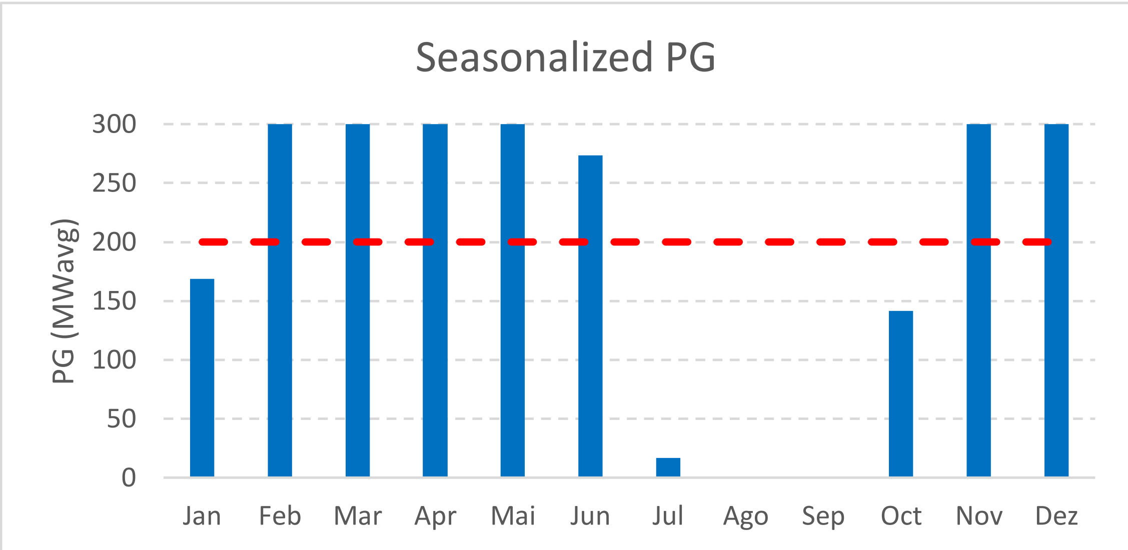


Figure 2.3: Example of Seasonalization

2.6 Energy Reallocation Mechanism

The Energy Reallocation Mechanism (ERM) was conceived to share among the participants, the financial risk associated to the energy commercialization of the centralized dispatched hydroelectric plants. The motivation to create to ERM comes from two main reasons, those are:

- The difference between effective energy generation, and the contractual obligations;

- The existence of hydro electrics plants in the same rivers cascade.

The first reason comes from the generation dispatch scheme adopted in the Brazilian energy market, that is made by the *tight pool* scheme, i.e., the Brazilian ISO (ONS) is responsible for each hydroelectric dispatch, aiming towards minimizing the whole system operational cost, that means, ultimately it is the ONS and not the companies, who decides the generation amount of the power plants. This ONS policy towards optimizing the system as a whole, and disregarding the local optimal decisions, may lead to some plants mismatching their contracts agreements, due to the lack of control of their energy generation. The second reason, is that there exists many hydroelectric plants along the same rivers cascade, hence the inflows are utilized by all the agents located in that area, and the owner of the plant the most uphill would have a control of all generation from under it.

The ERM guarantees that all the plants receives their levels of PG, as long as, the total ERM Generation is not under the total ERM PG, The ERM acts reducing this exposition risk, sharing it among all the plants in the mechanism. If the generation in the ERM is higher than the total PG, the participants have the right to receive an energy surplus, called *Secondary Energy*, allocated among all the ERM participants, proportionally to their PGs. On the other hand, if the total ERM generation is lower than the total ERM PG, each powerplant PG will be readjusted to match the total generation.

This mechanism is optional to the small hydroelectric plants, and compulsory to the big hydroelectric plants. More details about this mechanism, and how all the calculations behind it can be found in the Appendix.

2.7 Penalties

There are some penalties for the agents that sells more energy than its capability of providing, which is called an under-contracted situation. The penalty applied to the agent is valuated at the highest one among spot prices, and the *annual reference value* (ARV)[1], times the energy deficit of the month lacking energy. The CCEE is responsible for monitoring this energy gap.

2.8 Energy Contracts

In the Brazilian energy market, there are some different types of contracts, and the most common are the following:

- Quantity contracts;

[1]The ARV is calculated by ANEEL annually.

- Availability contracts;
- Swap contracts.

The Quantity contracts, as the name states, are the agreement of selling (or buying) a fixed amount (quantity) of energy between two parts, at a predefined period in time. They are the most common kind of contract established in the Brazilian energy market. The risk is assumed by the seller in this kind of contract.

The Availability contracts are based on the idea of renting the generation curve of the power plant. They are usually established between energy agents with thermal power plants. The risk is assumed to the buyer in this type of contract.

Finally, Swaps contracts, are not contracts in the strict sense of the word, but rather an instrument to hedge risk. It is an exchange of contracts, that may be, a temporal swap (exchange between energy maturity dates contracts), submarkets swaps (exchange between different Brazilian energy sub-regions), or sources swaps (exchange between different source generation power plants).

Chapter 3

Linear Programming and Risk Measures

3.1 Linear Programming

One of the goals of this work, is to translate the contracts and PG optimal allocation into a linear optimization problem. So the first step of this task is to define what is this kind of problems. Essentially, it is a subset of optimization problems, that can be fully described by linear relationships of the utility (or cost) function, and constraints.

A linear optimization problem can be written in the following canonical form:

$$\begin{aligned} \max_{x\in\Re^n} \quad & c^T x \\ \text{s.t} \quad & Ax \leq b \\ & x \geq 0 \\ & c \in \Re^n, A \in \Re^{m\times n}, b \in \Re^m \end{aligned}$$

In order to solve such a class of optimization problems, there exist two main algorithms capable of finding the optimal solutions: the Simplex Method, and the Interior Point Algorithm [31]. Both of these algorithms are capable of finding solutions that satisfies the Karush-Kuhn-Tucker conditions.

3.1.1 Karush-Kuhn-Tucker conditions

The Karush-Kuhn-Tucker conditions (KKT) are necessary and sufficient conditions (for linear problems) to find an optimal point. Consider a general optimization problem as follows:

$$\begin{aligned} \max_{x \in \Re^n} \quad & f(x) \\ \text{s.t} \quad & g_i(x) \leq 0, \forall\, 1 \leq i \leq m \\ & x \geq 0 \end{aligned}$$

Theorem 1

The KKT conditions state that if $f(x)$ is convex, and g_i is an affine function (which holds for linear optimization problems), then no other condition is needed to guarantee the global optimality of x^*.

The KKT conditions of this problem are described as the following equations:

$$\begin{aligned} \nabla f(x^*) &= \sum_{i=1}^{m} \mu_i \nabla g_i(x^*) \\ g_i(x^*) &\leq 0, \forall i\ 1 \leq i \leq m \\ \mu_i g_i(x^*) &= 0, \forall i\ 1 \leq i \leq m \\ \mu_i &\geq 0, \forall i\ 1 \leq i \leq m, \end{aligned}$$

where x^* are the optimal values.

3.2 Risk Measures - Introduction

The portfolio problem takes into account many scenarios realization, in order to make the best allocation decision, for this reason it is important to study and incorporate models that measures the associated risk, but before beginning to define risk measures, with the associated mathematical formulation, it is considered a simple example of a contract allocation in the energy market:

A hydroelectric company has an amount of 10 MWh to sell in energy contracts. It can be sold as a forward contract at price $P_s = 240€/MWh$ today, or at the spot price at the end of the month. However, the spot price is still unknown. In order to quantify this price, the company's analysts have made the projection listed in table 3.1. They have also plotted the Cumulative Distribution Function of the revenue in Fig.3.1, as if this energy would be entirely sold at the spot market, hence the revenue of the company would be straightforwardly given by the amount of energy sold times the spot price at the end of the month.

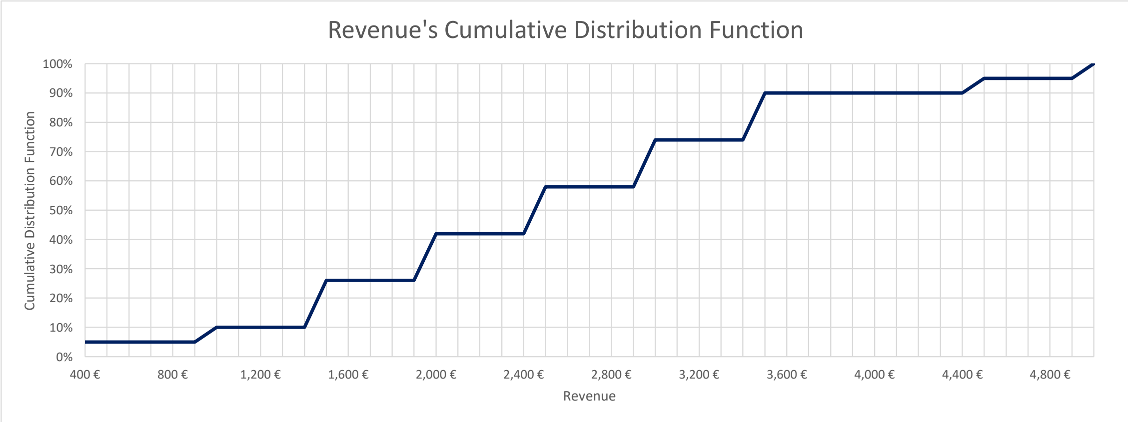


Figure 3.1: CDF of Revenue

Table 3.1: Hydroelectric company spot price projection

Spot Price (€/MWh)	Probability	Revenue (€)
40	5%	400
100	5%	1.000
150	16%	1.500
200	16%	2.000
250	16%	2.500
300	16%	3.000
350	16%	3.500
450	5%	4.500
500	5%	5.000

It can be seen from Fig. 3.1 that on one hand the revenue has a 5% probability of being 5.000€, but on the other hand, it has 5% probability of being 400 €. In this case, it is said that there is a 5% *risk* of gaining only 400 €. If the company chooses to sell it entirely in a forward contract, the revenue of 2.400€ is guaranteed. The question that arises is how to maximize the revenue taking in account the risk, and how to translate the concept of risk in a mathematical form.

3.3 Modern Portfolio Theory

In the matter of risk analysis, one of the first contributions is the markowitz mean-variance portfolio theory [4]. In his analysis, the risk is perceived as the correlation among different assets, as a result, in order to reduce the risk one should create a portfolio composed of negative correlated assets. Since the markowitz theory, an expressive number of risk measures and different risk analysis have been developed [32]. Among them, two of great importance are the Value-at-Risk (VaR) [6], and the Conditional-Value-at-Risk (CVaR) [7]. The risk measure utilized by markowitz (minimal variance) is not so adequate for the financial products in the Brazilian energy market, this is mainly due to the fact that all financial contracts have a positive correlation, influenced by the spot prices. Since the risk measures such as the VaR, and CVaR synthesize in one number the effective capital in risk, they are more suitable to the problem.

3.4 Value-at-Risk

The Value-at-Risk (VaR) summarizes the worst scenario over a target horizon that will not be exceeded with a given level of confidence [6]. It is a risk measure created by financial agents, and became a benchmark of risk in the nineties, but that remains highly used nowadays.

3.4.1 Informal Definition

The VaR at a confidence level (1-β)% is defined as the maximum revenue that will not exceed that threshold of the worst (β%) scenarios. In order to illustrate this concept, it is considered the same example from section 3.2. Analyzing the graph of the revenue's *Cumulative Distribution Function* in Fig. 3.1, at for instance, $\beta = 10\%$, it is possible to conclude that, the VaR at confidence level of 90% is 1.000 €, because at a 90% confidence level, in other words, in 90% of the cases, the worst possible revenue is 1.000€.

On the other way around, the VaR could be defined, by a similar (symmetrical) approach, as the maximum gain possible amongst all the worst scenarios. In the example, it is noted that for the 10% worst cases revenues, the best among them, as seen on Fig. 3.1, is the VaR, i.e., 1000€.

3.4.2 Mathematical Definition

In order to define the VaR mathematically, first consider the definition of the cumulative distribution function (Ψ), that is plotted in Fig. 3.1. The ordered ascending revenue is plotted in the x axis, while in the y axis is depicted the cumulative distribution function, which range goes from 0 to 100%, in mathematical terms:

$$\Psi(x, \alpha) = \int_{f(x,y) \leq \alpha} p(y) dy, \tag{3.1}$$

where x are the decision variables, $p(y)$ is the probability associated with the event y.

In order to clarify this function, some numerical evaluations of it, at the initial point x_0, that represents all the energy sold at the spot price are calculated as follows:

- $\Psi(x_0, 400) = 5\%$.
- $\Psi(x_0, 1000) = 10\%$.
- $\Psi(x_0, 1500) = 26\%$.

After defining the cumulative distribution function, it is possible to define the β VaR:

$$VaR_\beta = \alpha_\beta = \max\left\{\alpha \in \Re : \Psi(x, \alpha) \le \beta\right\} \tag{3.2}$$

In order to illustrate this concept, lets consider the example of section 3.2, where the scenarios are illustrated in Fig.3.1 and Table.3.1, in this situation, considering β equals 10%, the maximum revenue α that still satisfies $\Psi(x, \alpha) \le 10\%$ is 1.000€ , which implies in $\alpha_{10\%} = 1000$€, on other words, the 10% VaR is 1000 €.

Even though the VaR has become a benchmark as a risk measure, it lacks the capacity of considering extreme events, taking for instance our example, the VaR would not take into account the possibility of a 400€ revenue. Furthermore, it is not a coherent risk measure [33], and it is not linear, and as we are building a linear optimization problem, consequently it is still necessary to develop a more suitable risk measure.

3.5 Conditional-Value-at-Risk

The Conditional-Value-At-Risk (CVaR) also known as Mean Excess Loss, Mean Shortfall, or Tail Value-at-Risk, is defined as the weighted average of all values that lies between the VaR, and the worst possible scenario. It is a more conservative risk measure than the VaR [34].

In this work, the risk measure CVaR is chosen based on the following reasons:

1. It is linear [35].
2. It is a coherent risk measure. (fulfill all 4 axioms of coherence) [33].
3. It takes in account the skew and kurtosis of the revenue's distribution [7].

3.5.1 Informal definition

In order to illustrate the CVaR definition, the introductory example illustrated in Fig. 3.1 is considered, and the objective is to calculate the CVaR for a 90% confidence level. The CVaR is the weighted average of all the possible revenues that lies in between the 10% worst cases, which are directly read from table 3.1. In these

10% worst cases there are two scenarios: a 5% probability revenue of 400€ , and a 5% probability revenue of 1000€, thus the average revenue between these scenarios is 700€:

$$CVaR_{10\%} = \frac{400 \times 5\% + 1000 \times 5\%}{5\% + 5\%} = 700 \tag{3.3}$$

3.5.2 Mathematical definition

The above definition of the β CVaR for continuous time case is written mathematically as the following equation:

$$CVaR = \phi_\beta = \frac{1}{\beta} \int_{f(x,y) \leq \alpha_\beta(x)} f(x,y)p(y)dy, \tag{3.4}$$

where α_β is the β VaR, $f(x,y)$ is the revenue function, depending on x as the decision variable, and y is the unknown, defined in our case as the spot price(S_π).

In the discrete case the β CVaR is defined as:

$$\phi_\beta = \frac{1}{\beta} \sum_{f(x,y) \leq \alpha_\beta} f[x,y] * p[y] \tag{3.5}$$

However, it is useful to rewrite (3.5) as the following equation:

$$\phi_\beta = \alpha_\beta + \frac{1}{\beta} \sum (f[x,y] - \alpha_\beta)^- * p[y], \tag{3.6}$$

where $(\lambda)^-$ is the *negative truncation operator*, that is defined as:

$$\lambda^- = \begin{cases} \lambda, & \text{if } \lambda \leq 0 \\ 0, & \text{if } \lambda \geq 0 \end{cases} \tag{3.7}$$

In order to show that 3.6 is equivalent to 3.5, first three probability vs. revenue points' are plotted in Fig. 3.2.

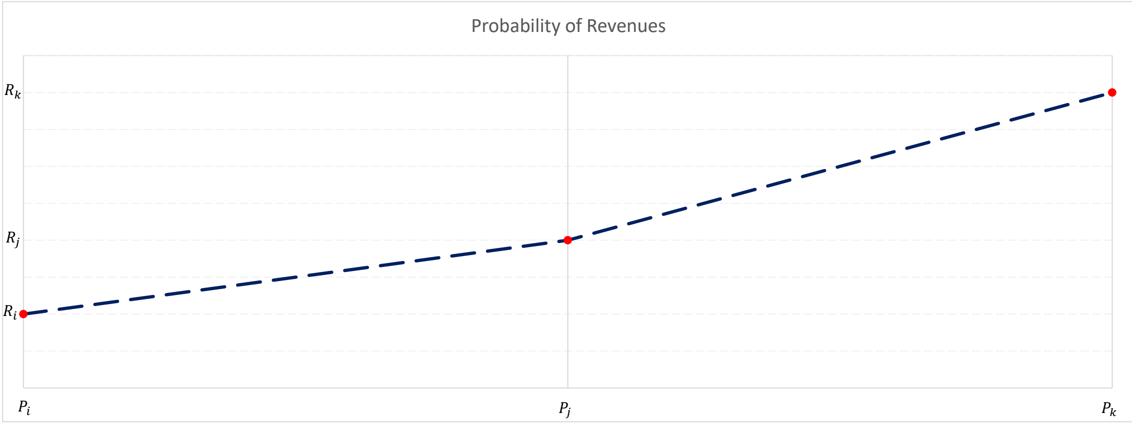


Figure 3.2: Example - Three probability X Revenue fictional points

Then lets define:

$$VaR_\beta = \alpha_\beta = x_j \tag{3.8}$$

From the first definition of the discrete CVaR 3.5], it is possible to write the CVaR as:

$$\phi_\beta = \frac{p_i x_i + p_j x_j}{p_i + p_j} \tag{3.9}$$

For these two points, it is easy to show that:

$$\phi_\beta = x_j + \frac{(x_i - x_j)^- p_j + (x_k - x_j)^- p_k}{p_i + p_j} = \frac{p_i x_i + p_j x_j}{p_i + p_j} \tag{3.10}$$

For n points, the formula is generalized:

$$\phi_\beta = \alpha_\beta + \frac{1}{\beta} \sum_{j=1}^{J} p_j (x_j - \alpha_\beta)^-, \tag{3.11}$$

Therefore the two forms of writing the β CVaR are equivalent.

One key aspect of (3.4) (or 3.5) is that in order to compute the CVaR, a previous knowledge of the VaR is necessary to know the feasible region of the integral ($f(x, y) \leq \alpha_\beta(x)$). However, it is possible to decouple the VaR from the CVaR, in such a way that the computation of the CVaR is done independently from the former. It will be shown how this can be done in the next section.

3.5.3 Independent CVaR Formulation

The aim of this section is to provide a way to compute the CVaR without the necessity of pre-calculating the VaR.

First it is defined the function that is called the F_β, which has an important role in demonstrating how to compute the CVaR independently from the VaR:

$$F_\beta(x,\alpha) = \alpha + \frac{1}{\beta}\sum_{\forall y} p[y] * (f[x,y] - \alpha)^- \tag{3.12}$$

The first statement to this proof, is that maximizing F_β with respect to α, the value obtained for $\alpha^* = \alpha_\beta$, is the VaR, mathematically:

$$\begin{aligned} &\max_\alpha\{F_\beta(x,\alpha)\} \\ &\alpha^* = \alpha_\beta \end{aligned} \tag{3.13}$$

Then a second statement that will also be proved is that, if the objective function is evaluated with this value of α^*, it would have the value of the CVaR, mathematically:

$$\phi_\beta(x) = F_\beta(x,\alpha_\beta) \tag{3.14}$$

The proof of these statements will be provided in two parts, one part for each one of the statements.

First part

First lets take the derivative of F_β with respect to α:

$$\frac{\partial F_\beta}{\partial \alpha} = 1 - \frac{1}{\beta}\sum_{\alpha \le f(x,y)} p_y \tag{3.15}$$

The KKT conditions for an optimal point of a maximization problem, without constraints is simply given by:

$$\nabla f = 0 \tag{3.16}$$

Writing the derivative of F_β with respect to α and equaling to zero :

$$\frac{\partial F_\beta}{\partial \alpha} = 1 - \frac{1}{\beta}\sum_{\alpha \le f(x,y)} p_y = 0 \iff \sum_{\alpha \le f(x,y)} p_y = \beta \iff \alpha^* = \alpha_\beta \tag{3.17}$$

It has been proven that to satisfy the KKT conditions (and thus find an optimal point), the sum of probabilities in the defined regions must be β ($\sum p_y = \beta$), recalling the definition of the VaR in 3.2, it is noticed that maximizing α turns the inequality into a equality precisely where, $\alpha = \alpha_\beta$, otherwise speaking, the α that maximizes the problem is the β VaR (α_β).

Second part

Recall the definition of the function F_β:

$$F_\beta(x,\alpha) = \alpha + \frac{1}{\beta}\sum_{y=1}^{Y}(f(x,y)-\alpha)^- p_y \tag{3.18}$$

Plugging $\alpha = \alpha_\beta$ into 3.18:

$$F_\beta(x,\alpha_\beta) = \alpha_\beta(x) + \frac{1}{\beta}\sum_{y=1}^{Y}(f(x,y)-\alpha_\beta(x))^- p_y \tag{3.19}$$

The sum in 3.19 can be rearranged, eliminating the necessity of the negative truncate operator, by changing the sum region:

$$F_\beta(x,\alpha_\beta) = \alpha_\beta + \frac{1}{\beta}\sum_{y=1}^{Y}(f(x,y)-\alpha_\beta(x))^- p_y = \alpha_\beta + \frac{1}{\beta}\sum_{f(x,y)\le\alpha_\beta(x)}(f(x,y)-\alpha_\beta(x))p_y \tag{3.20}$$

Separating the two elements ($f(x,y)$ and $\alpha_\beta(x)$)in the sum, α_β can be taken outside the sum as it does not depend on y:

$$\begin{aligned} F_\beta(x,\alpha_\beta) &= \alpha_\beta + \frac{1}{\beta}\sum_{f(x,y)\le\alpha_\beta(x)}(f(x,y)-\alpha_\beta(x))p_y \\ F_\beta(x,\alpha_\beta) &= \alpha_\beta + \frac{1}{\beta}\sum_{f(x,y)\le\alpha_\beta(x)}(f(x,y))p_y - \alpha_\beta(x)\times\sum_{f(x,y)\le\alpha_\beta(x)}(p_y) \end{aligned} \tag{3.21}$$

Recall that α_β may be defined as the (maximum) ordered revenue from worst to the best scenarios, in which the cumulative probability function will not exceed β, that means, the sum of all the probabilities in the region $f(x,y)\le\alpha_\beta$ must be β by definition, i.e.:

$$\sum_{f(x,y)\le\alpha_\beta(x)} p_y = \beta \tag{3.22}$$

Then, plugging 3.22 into equation 3.21:

$$F_\beta(x,\alpha_\beta) = \alpha_\beta + \frac{1}{\beta}\sum_{f(x,y)\le\alpha_\beta(x)}(f(x,y))p_y - \alpha_\beta(x)\times\beta \tag{3.23}$$

Recalling the CVaR definition:

$$\phi_\beta = \frac{1}{\beta}\sum_{f(x,y)\le\alpha_\beta(x)}(f(x,y))p_y \tag{3.24}$$

The CVaR definition can replace some of the terms in the equation:

$$F_\beta(x, \alpha_\beta) = \alpha_\beta(x) + \frac{1}{\beta}(\beta\phi_\beta - \alpha_\beta(x)\beta) = \phi_\beta(x) \tag{3.25}$$

The proof of the formulation is concluded, and from now on it is possible to compute the CVaR as the maximization of the $F_\beta(x, \alpha)$.

3.6 CVaR as a constraint

3.6.1 Equivalent Efficient Frontiers

In the previous sections it has been demonstrated how to compute the CVaR as an optimization problem, however, in this work we are interested in maximizing the expected revenue of the company, while considering the risk as a constraint. In this section it is demonstrated how to include the CVaR as a constraint to our optimization problem.

Consider three different optimization problems, i.e., P_1, P_2 and P_3, defined as the following:

$$\begin{aligned} &P_1\colon \max\ \{\phi_\beta(x) + \lambda R(x), x \in X\} \\ &P_2\colon \max\ \{\phi_\beta(x), R(x) \geq \rho, x \in X\} \\ &P_3\colon \max\ \{R(x), \phi_\beta(x) \geq \omega, x \in X\}, \end{aligned} \tag{3.26}$$

where $R(x)$ is the expected revenue function, ϕ_β is the β CVaR, ρ is the minimum revenue allowed, and ω is the minimum risk (CVaR) allowed.

It will be demonstrated that the maximization of these problems, generate the same efficient frontier. In order to show this statement, initially the KKT conditions of the three problems above are written, i.e, KKT1, KKT2, and KKT3:

$KKT1$:

$$\nabla\phi_\beta(x^*) + \lambda\nabla R(x^*) = 0$$

$KKT2$:

$$\begin{aligned} \nabla\phi_\beta(x^*) + \mu_2\nabla R(x^*) &= 0 \\ \rho - R(x^*) &\leq 0 \\ \mu_2(\rho - R(x^*)) &= 0 \end{aligned}$$

$KKT3$:

$$\begin{aligned} \nabla R(x^*) + \mu_3\nabla\phi_\beta(x^*) &= 0 \\ \omega - \phi_\beta(x^*) &\leq 0 \\ \mu_3(\omega - \phi_\beta(x^*)) &= 0 \end{aligned}$$

Consider x^* to be the solution of P_1, then by definition it has to satisfy the

KKT_1. It should be noted that this solution (x^*) also satisfies the KKT_2 with $\rho = R(x^*)$ and $\mu_2 = \lambda$. This is easily verified by plugging these values in the KKT_2, then comparing KKT_1 with KKT_2, the conditions will be exactly the same, since the last two expressions of KKT_2 will be zero, and as $\mu_2 = \lambda$, the first expression of the KKT_2 will be equal to the KKT_1.

Notice that x^* satisfies the KKT_3 by definition. Besides that, considering $\omega = \phi_\beta(x^*)$, and $\mu_3 = \lambda$, these conditions will turn the last two expressions of KKT_3 to zero, and consequently KKT_3 will be equivalent to KKT_1.

Since all the solutions are satisfied with x^*, moreover when $\omega = \phi_\beta$ and $\rho = R(x^*)$, the efficient frontier of P_1, P_2 and P_3 coincides. For that reason we conclude that all the problems have the same efficient frontier, when the optimal points are exactly at the constraint equality point ($\omega = \phi_\beta$, and $\rho = R(x^*)$), and there is only one optimal solution to them.

3.6.2 Replacing the constraint ϕ_β by F_β

It was proved that the problems P_1, P_2, and P_3 converge to the same optimal outcome. However, the final formulation of the risk constraints will be written in terms of F_β and not ϕ_β. Hence it must be shown that ϕ_β can be replaced by F_β without loss of generality. That is done by recalling the results of the independent CVaR formulation in 3.13:

$$\begin{aligned} \phi_\beta(x) &= \max_\alpha\{F_\beta(x,\alpha)\} \\ \alpha^* &= \alpha_\beta, \end{aligned} \tag{3.27}$$

which states that, as $F_\beta(x,\alpha)$ is optimized, the optimal point results in with $\alpha^* = \alpha_\beta$ and $F_\beta(x,\alpha_\beta) = \phi_\beta$. Since in this scenario, we are optimizing the (equivalent) problem, consequently at the optimal points:

$$\begin{aligned} &F_\beta(x^*,\alpha^*) = \phi_\beta(x^*) \\ &\alpha^* = \alpha_\beta \end{aligned} \tag{3.28}$$

3.7 Linearization

The negative truncation operator, λ^- (which is part of the F_β function), yields the maximum value between zero, and the value of the inputs. It is mathematically defined by:

$$\lambda^- = \begin{cases} \lambda, & \text{if } \lambda \leq 0 \\ 0, & \text{if } \lambda \geq 0 \end{cases} \tag{3.29}$$

The important thing to note is that is piecewise linear, and it would not be possible to include this definition directly into a linear optimization problem form. Nevertheless, it is possible to make use of a mathematical manipulation to re-write it as a set of linear constraints.

The proof is made by first recalling the definition of F_β:

$$F_\beta = \alpha + \frac{1}{\beta} \sum (f[x,y] - \alpha)^- p(y) \tag{3.30}$$

Then the negative truncate operator may be replaced, by including a new variable, and two new constraints, as the following:

$$\begin{aligned} &F_\beta = \alpha + \frac{1}{\beta} \sum z(y) p(y) \\ &z(y) \leq 0 \\ &z(y) \leq f[x,y] - \alpha \end{aligned} \tag{3.31}$$

In order to prove that these two alternative ways, 3.30 and 3.31, of writing F_β states the same outcome, it must be analyzed the two cases that may occur:

1. $f[x,y] - \alpha \leq 0 \implies z \leq f[x,y] - \alpha$
2. $f[x,y] - \alpha \geq 0 \implies z \leq 0$

The z values are being maximized in the objective function, consequently, the search for the maximum z turns the inequalities into equalities:

1. $f[x,y] - \alpha \leq 0 \implies z = f[x,y] - \alpha$
2. $f[x,y] - \alpha \geq 0 \implies z = 0$

The outcomes of both equations are equivalent when maximizing F_β, and effectively it is possible to write the problem as linear optimization problem.

3.8 Final Portfolio Problem Formulation

Finally, all the formulations described previously in this section are grouped into one single optimization problem:

$$
\begin{aligned}
&\max\ R(x) \\
&\text{s.t} \\
&F_\beta(x,\alpha) = \alpha + \frac{1}{\beta}\sum_{j=1}^{NS} p_j \times z_j \geq \omega_0 \\
&z_j + \alpha \leq f(x, y_j), \forall j \\
&z_j \leq 0, \forall j
\end{aligned}
\tag{3.32}
$$

Chapter 4

Problem Formulation

4.1 Introduction

In this chapter, all the decision options and constraints of the hydroelectric generation company are brought together in mathematical terms, in the form of an optimization problem. All the concepts discussed, such as the revenue, risk, and contracts are now variables of this problem.

4.2 Mathematical Formulation

All the variables of the problem are defined in Table 4.1. More details about these variables will be presented in the following sections.

Table 4.1: Variables Definition

Variable	**Description**	**Units**
P_m^s	Selling price of forward at month m	€/MWh
P_m^b	Buying price of forward	€/MWh
x_m^s	Forward Selling Quantity	MWh
x_m^b	Forward Buying Quantity	MWh
$E[S_\pi]_m$	Expected spot price	€/MWh
GSF_m	Generation scaling factor	–
α	Value at Risk	€
β	Conditional Value at Risk tail	€
NS	Number of scenarios	–
p_j	Probability of scenario j	–
z_j	Auxiliary variable to compute the CVaR associate with scenario j	€
ω_0	Minimum revenue at β%	€
$f(x, y_j)$	Revenue as a function of forward and spot prices	€
h_m	Hours of month m	h
PG_r	Regulated Physical Guarantee	MWh
PG_m	Physical Guarantee at month m	MWh
P_{HE}	Hydroelectric power	MW
λ_m	Monthly energy balance	MWh
X_m^s	Maximum amount of selling forwards	MWh
X_m^b	Maximum amount of buying forwards	MWh
γ_m	Penalty price at month m	€/MWh

Hydroelectric generation companies are allowed to buy and sell energy at prefixed prices (buying price P_b and selling price P_s) at the beginning of a period (one month or more), by making forward contracts, or at the spot price S_π at the end of each month (at the market clearing). The total revenue of the company is straightforwardly given by the difference between the total energy sold at each price and the total energy bought at each price. The total energy available to be sold by the company is given by its effective PG plus its total buying forwards contracts x_b. The available energy has to be sold in the market by establishing selling forward contracts x_s at a prefixed price P_s at the beginning of the period, or at price S_π at the end of each month. The expected revenue is then given by

$$Rev = P_s \times x_s - P_b \times x_b + S_\pi \times (PG \times GSF + x_b - x_s) \tag{4.1}$$

Simultaneously the generation company would also like to control its risk at a minimum tolerable level. This control will be formulated as a constraint of the problem made by the CVaR β%. It can be interpreted as the average β% worst cases net revenue. The formulation of the CVaR as a linear constraint is explained in [35].

An important remark in this market is that there exists a penalty due to lack of *energy coverage*, consequently if the monthly balance is negative, the company will have to pay a penalty proportional to this amount (λ). This penalty is valued at the highest one among the (expected) spot price and the *annual reference value* (ARV)[1]. It is important to clarify, that the optimal decision may choose to pay the penalty (and leverage the quantity of selling energy) in a time period, if the optimal problem find it would bring a higher revenue than otherwise.

$$\gamma = max(E[S_\pi], ARV) \tag{4.2}$$

Additionally, the company must respect some regulatory limits, given in [1] and [3], these are:

- The annual average sum of the monthly PG seasonalized (PG_m) can't be bigger than the regulated PG (PG_r).
- The monthly PG seasonalized can't be higher than the hydroelectric power (P_{HE}).
- The hydroelectric company must have 100% contracted energy each month ($\lambda_m \geq 0$).

From the above definitions and equations, the optimization problem is defined as the following equations:

[1] The ARV is calculated by ANEEL annually

$$
\begin{aligned}
& max \sum_{m=1}^{12} P^s_m \times x^s_m - P^b_m \times x^b_m + \\
& E[S_\pi]_m \times (GSF_m \times PG_m + x^b_m - x^s_m) - \gamma_m \lambda_m && (4.3)\\
& s.t \\
& \alpha + \frac{1}{\beta} \times \sum_{j=1}^{NS} p_j z_j \geq \omega_0 && (4.4)\\
& z_j \leq 0, \forall j \; 1 \leq j \leq NS && (4.5)\\
& z_j + \alpha \leq f(x, y_j), \forall j \; 1 \leq j \leq NS && (4.6)\\
& 0 \leq PG_m \leq P_{HE} \times h_m, \forall m \; 1 \leq m \leq 12 && (4.7)\\
& \frac{1}{12} \sum_{m=1}^{12} PG_m = PG_r && (4.8)\\
& \lambda_m \geq x^s_m - x^b_m - GSF \times PG_m, \forall m \; 1 \leq m \leq 12 && (4.9)\\
& \lambda_m \geq 0, \forall m \; 1 \leq m \leq 12 && (4.10)\\
& 0 \leq x^b_m \leq X^b_m, \forall m \; 1 \leq m \leq 12 && (4.11)\\
& 0 \leq x^s_m \leq X^s_m, \forall m \; 1 \leq m \leq 12 && (4.12)
\end{aligned}
$$

The objective is to maximize the annual expected revenue defined in (4.3). The under-script m indicates that a variable is referenced to the month m. In (4.4), (4.5) and (4.6) the linearized CVaR constraints are described. The minimum revenue ω_0 should be defined by the company's risk aversion in the $\beta\%$ worst cases. In (4.7) it is defined the maximum amount of PG that a generator may allocate in a month, which is equal to the electric generator's power. In (4.8) it is defined another regulatory restriction in the seasonalization process, i.e, the average of the annual PG must respect the regulatory value (PG_r) defined by EPE and ANEEL. In (4.9) and (4.10) it is defined the *regulatory coverage* conditions, that are: the generator company may not sell more than it will buy (and generate) in a whole month, thus that the average monthly balance must be positive. In (4.11) and (4.12) it is defined the maximum amount of energy that can be bought and sold on forward contracts respectively, those limits may be chosen by the agent, or they can be a constraint in terms of the market liquidity.

The problem of optimizing the company's revenue taking into account the risk control, is written as a linear programming problem. In order to deal with this kind of problems, it is possible to proceed by using linear solvers such as the simplex, or the interior point algorithm to solve for the optimal contracts, and PG allocation. In the next chapter this formulation will be used to solve some examples of optimal allocation during a year for a hydroelectric generation company.

4.3 Simple Application

In this section, example from chapter 3 is revisited as an optimization problem to be solved mathematically. Recalling some assumptions of the example: it is a single month problem, the hydroelectric company cannot buy forwards, it can only sell forwards (up to 10MWh) at 240 €/MWh, furthermore this scenario has a fixed $GSF = 1$, and $PG = 10MWh$, finally there are no penalties due to lack of energy. Lets consider first the case where $\omega_0 = 1000$ €, then the case where $\omega_0 = 2000$€, for both cases $\beta = 10\%$.
Writing these conditions in the form of an optimization problem:

$$\begin{aligned}
& max\ 240 \times x_s + \\
& 254.5 \times (10 - x_s) && (4.13)\\
& \text{s.t} \\
& \alpha + \frac{1}{0.1} \times (5\% \times z_1 + 5\% \times z_2 + \\
& 16\% \times z_3 + 16\% \times z_4 + \\
& 16\% \times z_5 + 16\% \times z_6 + \\
& 16\% \times z_7 + 5\% \times z_8 + 5\% \times z_9) \geq \omega_0 && (4.14)\\
& z_j \leq 0, \forall j = 1...9 && (4.15)\\
& z_1 + \alpha \leq 240 \times x_s + 40 \times (10 - x_s) && (4.16)\\
& z_2 + \alpha \leq 240 \times x_s + 100 \times (10 - x_s) && (4.17)\\
& z_3 + \alpha \leq 240 \times x_s + 150 \times (10 - x_s) && (4.18)\\
& z_4 + \alpha \leq 240 \times x_s + 200 \times (10 - x_s) && (4.19)\\
& z_5 + \alpha \leq 240 \times x_s + 250 \times (10 - x_s) && (4.20)\\
& z_6 + \alpha \leq 240 \times x_s + 300 \times (10 - x_s) && (4.21)\\
& z_7 + \alpha \leq 240 \times x_s + 350 \times (10 - x_s) && (4.22)\\
& z_8 + \alpha \leq 240 \times x_s + 450 \times (10 - x_s) && (4.23)\\
& z_9 + \alpha \leq 240 \times x_s + 500 \times (10 - x_s) && (4.24)\\
& 0 \leq x_s \leq 10 && (4.25)
\end{aligned}$$

In this example, the average spot price is 254.5€/MWh, as a result, selling all the energy at the spot market would result in a bigger average revenue compared to selling all its energy in forwards (at 240€/MWh). For that reason the risk neutral decision would be to stay completely in the spot market. On the other side, by selling forwards the company has a good tool to hedge the risk of lower spot prices. A complete hedge would sell all its energy in forwards, and guarantee a revenue 2400€.

First Case:

In the first case, the solutions yield an average revenue of $< R >= 2519$€, while the selling forwards quantity are $x_s = 1.76$MWh, that means the decision is to sell the remaining energy $(10 - 1.76 = 8.24$MWh) at the spot price (market clearing). However, these 8.24MWh includes an intrinsic risk, on the other hand, the risk constraint established in this case has to guarantee a revenue (CVaR 10%) of 1000€, in the average worst 10% cases, which are: $P_s = 40$€/MWh (with a 5% associated probability) and $P_s = 100$€/MWh (with a 5% associated probability), as both of these scenarios have equal probability of 5%, the average worst price would be $< P_s >= 70$€/MWh , subsequently this worst CVaR price times the energy at the spot market results in $70 \times 8.24 = 578$€, plus the forward revenue $1.76 \times 240 = 422$€, finally the sum of both these revenues guarantees the risk constraint of 1000€.

Second Case:

In the second case, the solutions yields an average revenue of $< R >= 2434$€, and the selling forwards quantity are $x_s = 7.64$MWh, that means the decision is to sell the remaining energy $10 - 7.64 = 2.36$MWh at the market clearing. However, these 2.36MWh have a risk, and the constraint established in this case has to guarantee a revenue of 2000€. The average worst 10% price is again $< P_s >= 70$€/MWh, hence the worst CVaR price times the energy at the spot price $70 \times 2.36 = 166$€, plus the selling forwards revenue $7.64 \times 240 = 1834$€ equals to 2000 €.

As the risk aversion goes up, the quantity of selling forwards also goes up as illustrated in Fig. 4.1, in order to guarantee that hedge of minimum revenue by selling the energy in forward contracts. On the other hand if the agent would like to take more risk, the decision would be to stay in the spot market to increase its average revenue, since the average spot price is higher then the current forward prices as illustrated in Fig. 4.2.

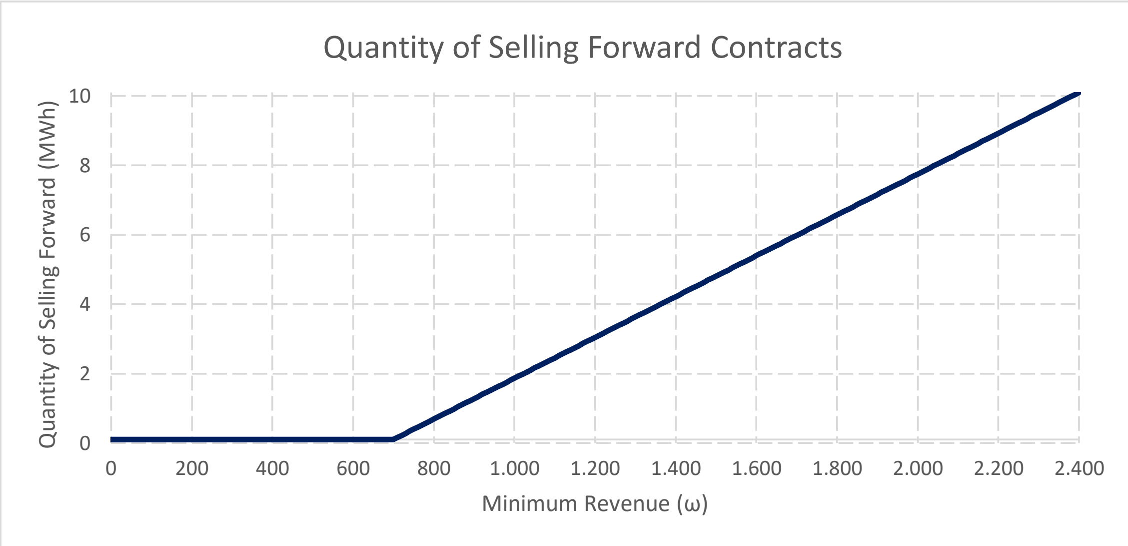


Figure 4.1: Quantity of selling forwards varying with the risk aversion.

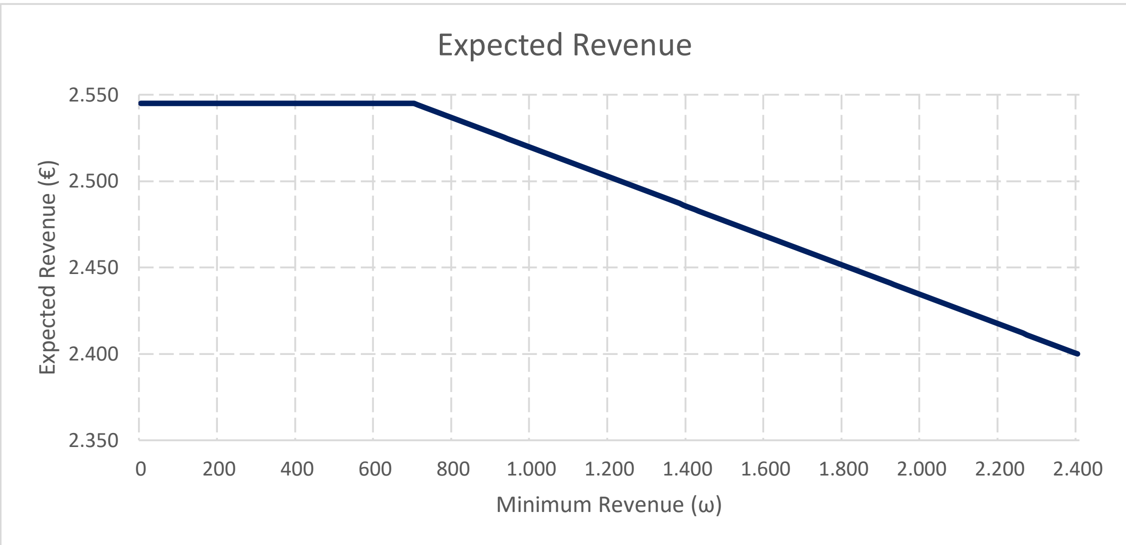


Figure 4.2: Expected Revenue varying with the risk aversion.

4.4 General overview of the optimization tool

In this section, in order to clarify the use of the optimization tool, an example of using the optimization tool during a year is depicted in Fig. 4.3. In December the hydro company has to define its PG of the next year, as well as the decision on how to buy and sell the forwards contracts of the following year. However, in order to effectively use the model at that time, the agents will have to provide the GSF projection, spot prices projections (S_π), and analyze the inputs from the market of the current energy contracts prices to the following months (P^s, P^b). Then in a next time step (the next month for instance, i.e., January), the decision will consist on how to optimize only the contracts, by making some modifications, since the PG is already defined and it cannot be modified, however as the vision of the future changes, i.e., spot prices projections and energy contracts prices vary, the portfolio may have some adjustments in order to be at a new optimal point, more suitable to the present date. This procedure may be repeated at any time in the future with the updates of the necessary parameters.

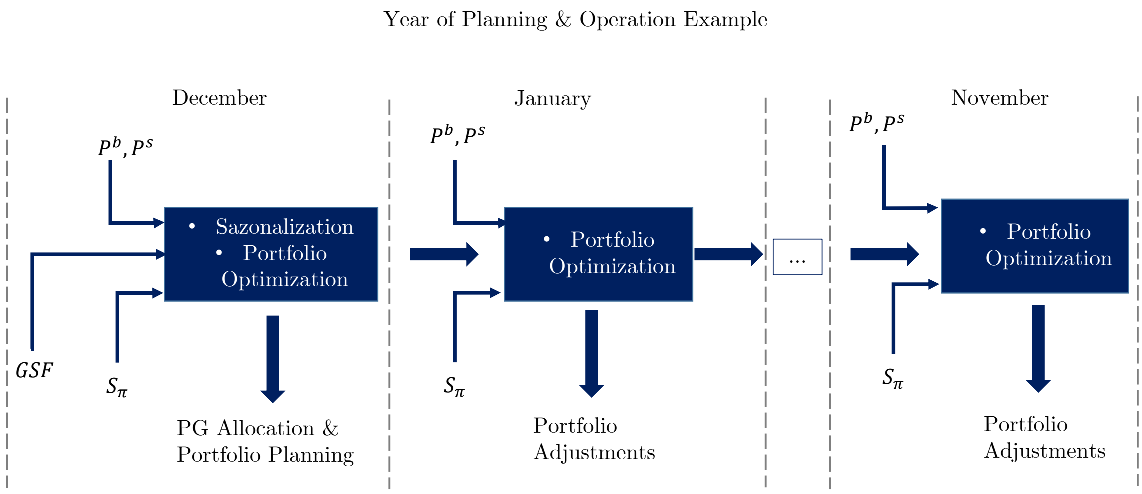


Figure 4.3: Example of using the optimization tool during the year

Another way of using the optimization tool during the year, is to make adjustments due to future evolution of premises according to a new market dynamic (which updates the P^s and P^b), and also due to new premises on the spot prices projection (S_π), the last portfolio position (x^s and x^b) is hence updated by these new inputs. This flow of actions is depicted in a block diagram in Fig. 4.4.

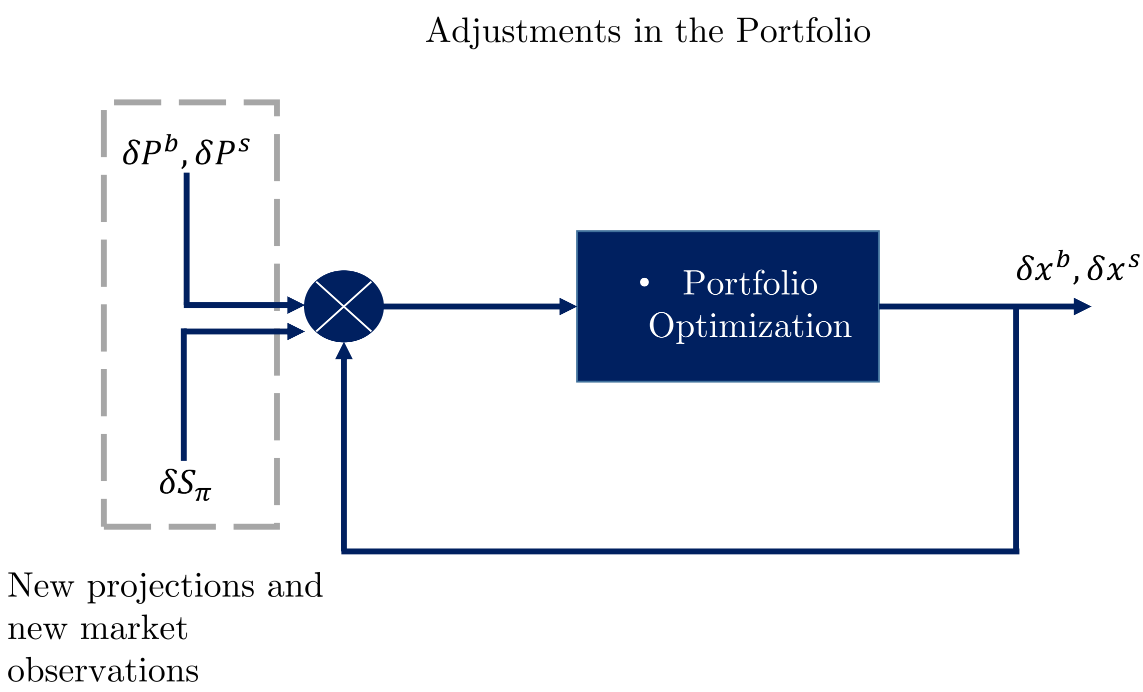


Figure 4.4: Block diagram of optimization tool

Chapter 5

Case Study

5.1 Introduction

In this section the portfolio of a 200MWavg[1] physical guarantee hydroelectric company is optimized. The available forwards contracts (by end of December 2018) are illustrated in table 5.1. Table 5.2 shows the projection of spot prices along the next year, while table 5.3 shows the associated probability with the scenarios (low, medium, and high). Finally in table 5.4 it is defined the maximum amount of selling/buying forwards contracts available.

In each case studied, two different approaches to the problem will be compared. The first one, is the risk neutral, in which CVaR constraints are not taken in account, put differently, the problem will be formulated without (4.4) to (4.6). In the second approach the maximum risk constraint (CVaR 10%) possible is included(defined as the maximum ω_0 that allows the problem to be feasible). The results are compared, and the pareto efficient frontier is plotted, showing all the points between the two approaches. The pareto efficient frontier is plotted by creating a linear interpolation of the optimal portfolio solutions that lies in between the risk neutral approach, and the maximum risk averse approach. The motivation of this frontier is to show the risk and return trade-off of every optimal portfolio solution.

On Case I it is considered only the contracts' optimal allocation, while on Case II it is considered the contracts and PG optimal allocation. Finally, on Case III, it is considered the contracts, and PG allocation with a different GSF (sensitivity study).

[1] Average of the energy along the month (total MWh divided by the hours of month).

Table 5.1: Forwards Prices

Date	**Buy(€/MWh)**	**Sell(€/MWh)**
January	38.1	38.2
February	52.3	52.5
Mars	52.7	54.5
2nd Trimester	53.8	54.5
2nd Semester	55.1	56.1

Table 5.2: Spot Price Projection (€/MWh)

Scenario	**January**	**February**	**Mars**	**2nd Trimester**	**2nd Semester**
Low	23.0	31.3	36.9	41.9	40.4
Medium	40.6	58.0	66.6	67.9	72.7
High	64.1	86.0	94.1	105.6	100.3

Table 5.3: Probability of Spot Price Projection

Scenario	**January**	**February**	**Mars**	**2nd Trimester**	**2nd Semester**
Low	25%	25%	25%	25%	25%
Medium	50%	50%	50%	50%	50%
High	25%	25%	25%	25%	25%

Table 5.4: Limits of forwards contracts (MWavg)

Type	**January**	**February**	**Mars**	**2nd Trimester**	**2nd Semester**
Buy	100	100	100	100	100
Sell	200	200	200	200	200

5.1.1 Case I

A decision of a flat physical guarantee is assumed along the year, thus only the contracts' allocation is optimized. The physical guarantee is subject to the GSF of the previous year (2018), and the *effective* PG (the PG multiplied by the GSF) is illustrated in Fig. 5.1.

In the risk neutral approach (illustrated in Fig. 5.2.), the optimized decision is to buy the maximum every month, while not selling anything as a forward contract.

The risk averse approach (illustrated in Fig. 5.3) decides to buy forwards mainly on the first semester, on the other hand, it decides only to sell a single forward contract in January. The explanation is due to a high expected value of the spot price, in order to, at the market clearing, the balance be positive an receive the energy difference at the spot price.

We finish this case by plotting the pareto efficient frontier as illustrated in Fig. 5.4.

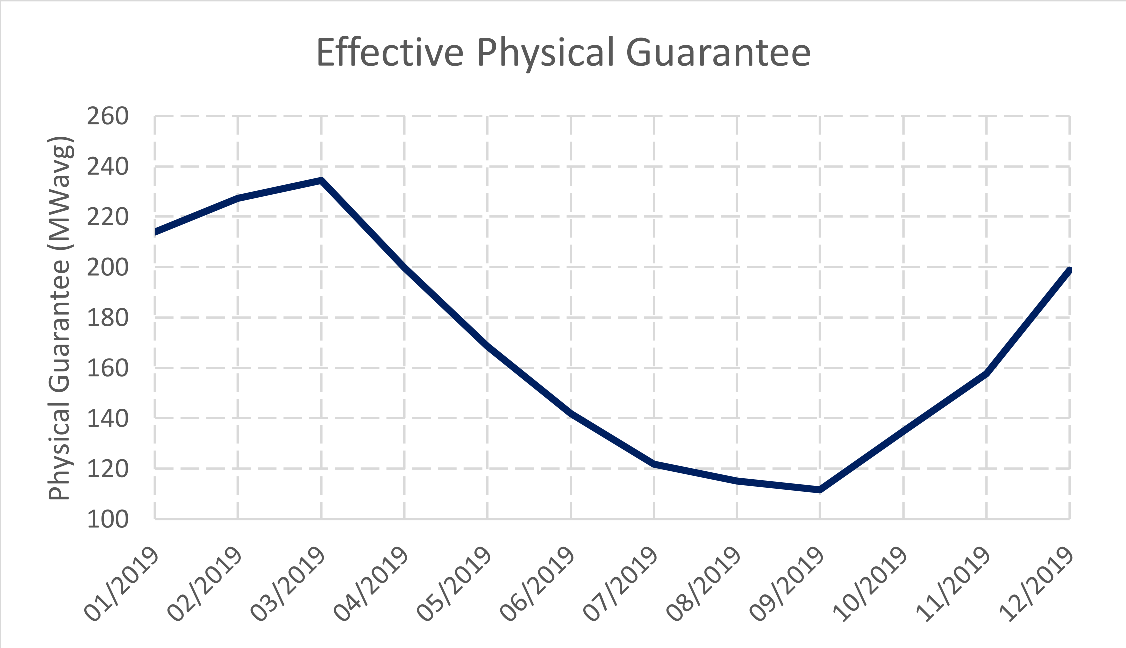


Figure 5.1: Case I - Seasonalized PG with GSF.

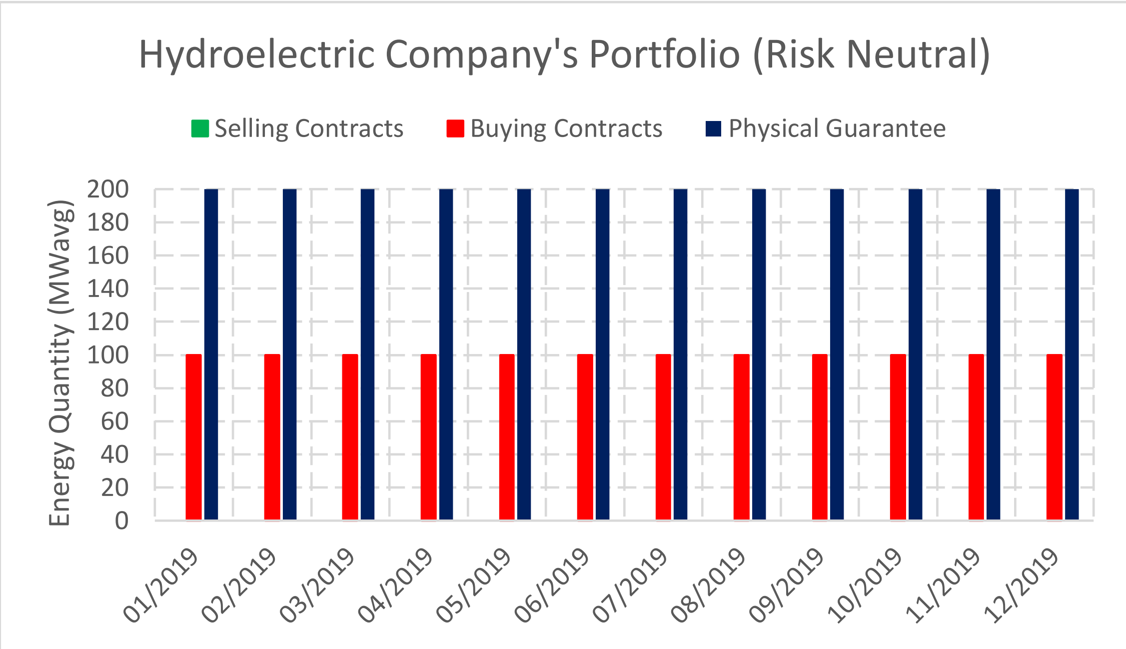


Figure 5.2: Case I - Risk Neutral Portfolio.

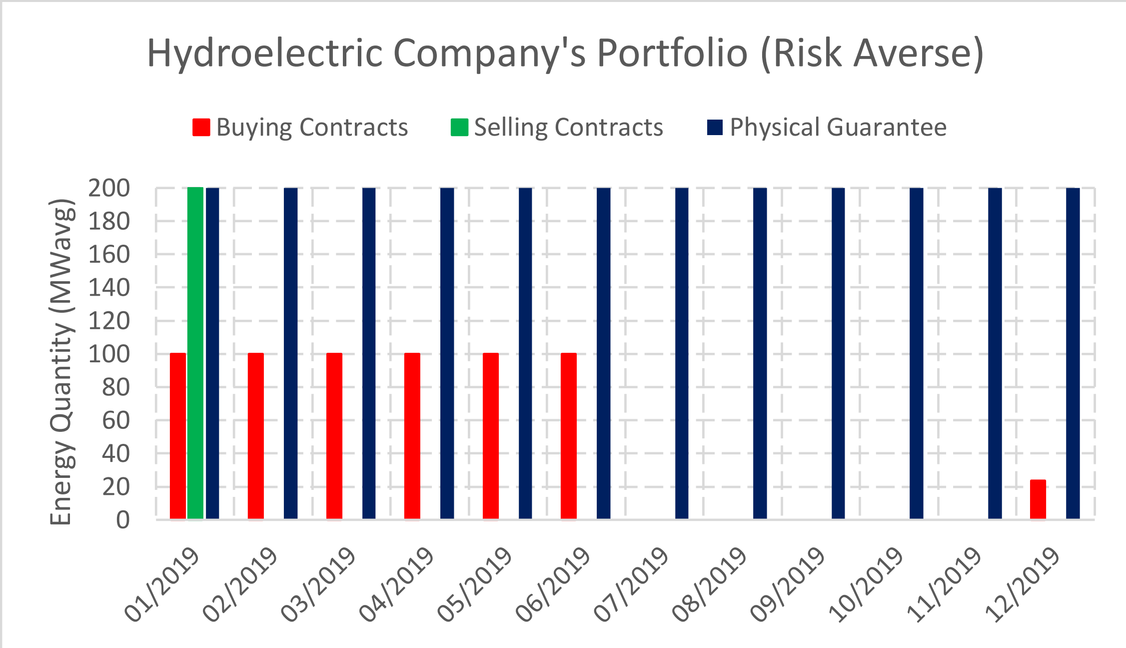


Figure 5.3: Case I - Risk Averse Portfolio.

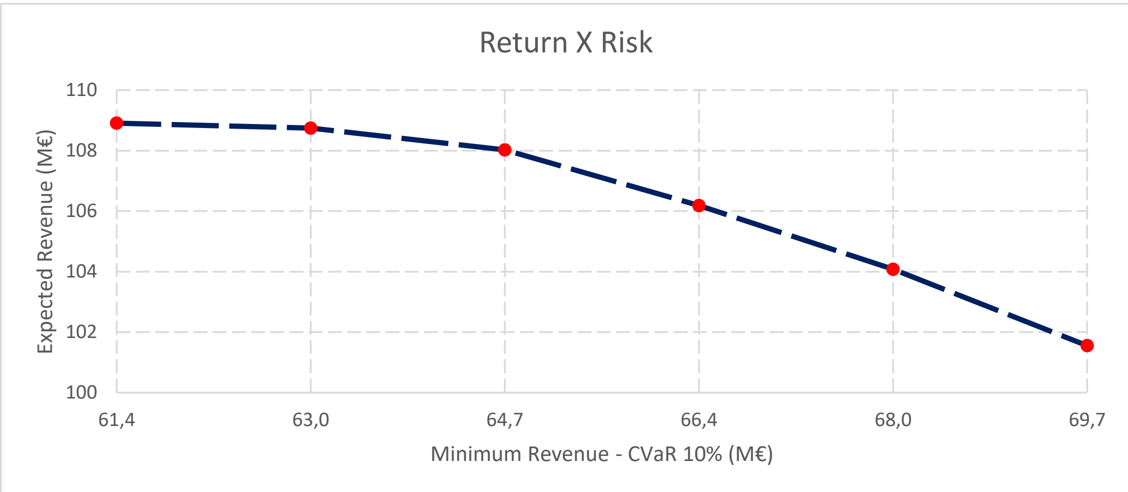


Figure 5.4: Case I - Pareto Efficient Frontier.

5.1.2 Case II

In this case the possibility to seasonalize is include, by considering the PG as a decision variable.

In the risk neutral approach (illustrated in Fig. 5.5), the decision concerning the forwards contracts would be the same from Case I, i.e., buying the maximum every month, while not selling anything as a forward contract. The decision to seasonalize the PG in this case, is to allocate the PG in almost all of the first semester, and in the last three months.

In the risk averse approach (illustrated in Fig. 5.6), the decision differs from Case I. It decides to buy forwards in the first trimester, and in the second semester, while selling a single forward in January. The PG allocation is now located mainly in the first semester, and the last two months of the year.

It can be noticed that the return in this case, depicted in Fig. 5.7, is higher than Case I, because it is viable to maximize the profit from PG allocation.

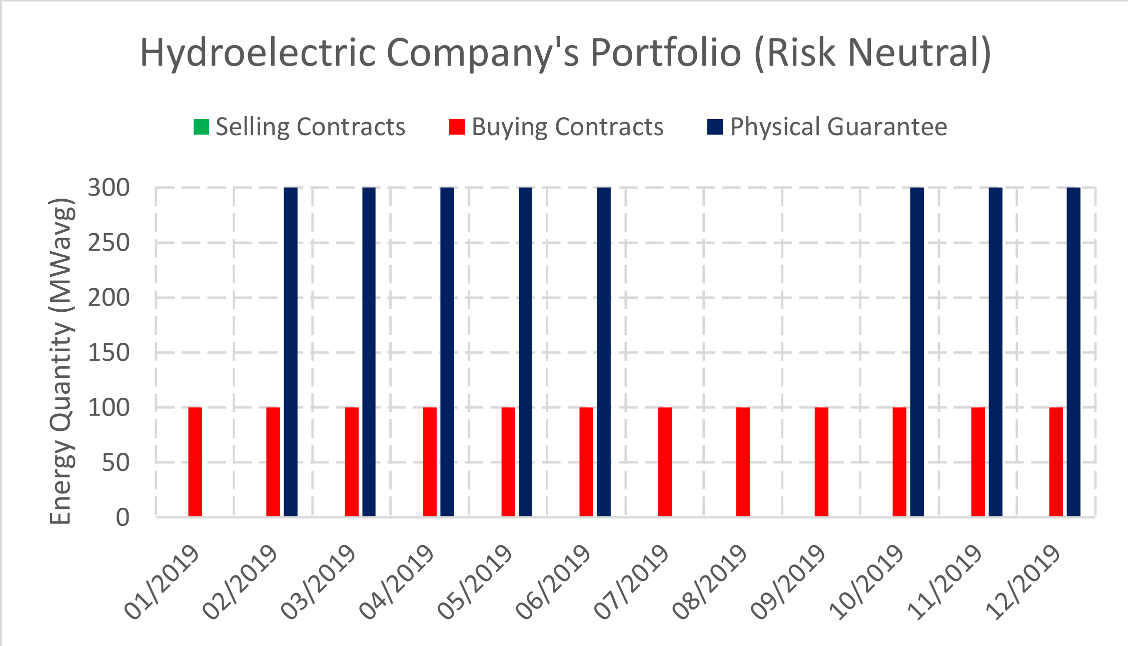


Figure 5.5: Case II - Risk Neutral Portfolio.

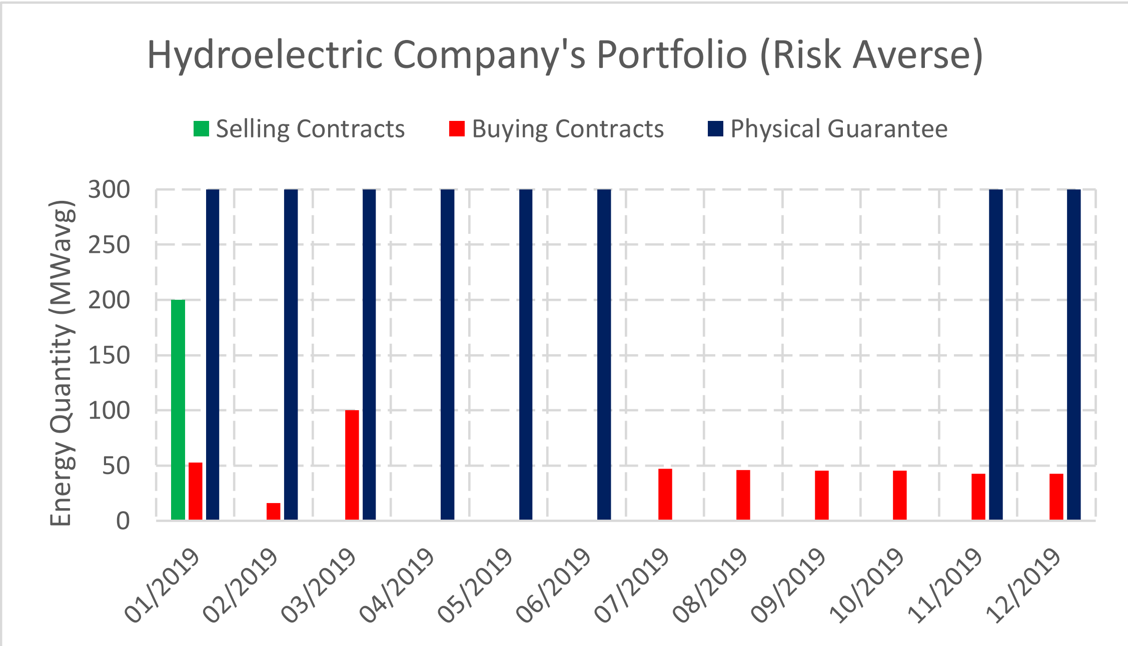


Figure 5.6: Case II - Risk Averse Portfolio.

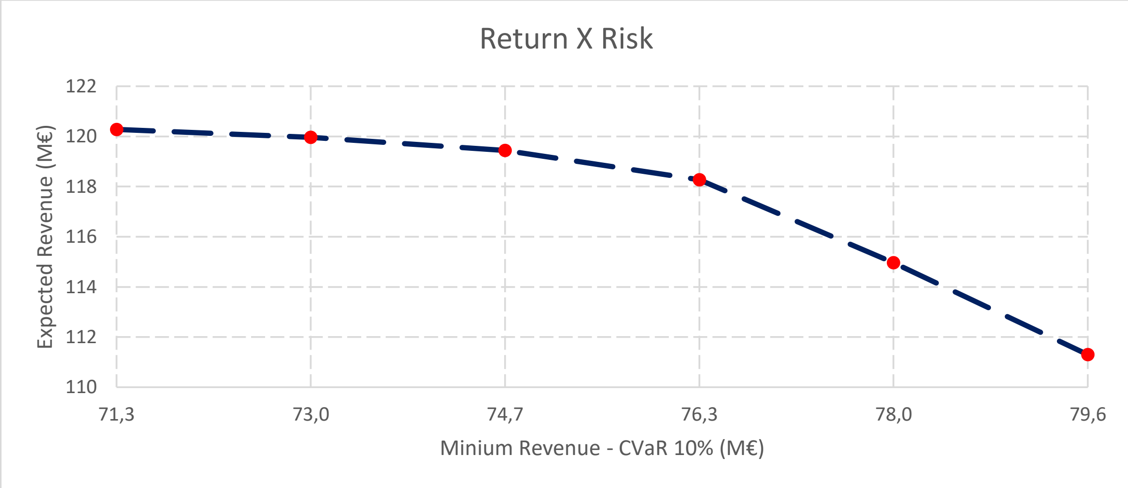


Figure 5.7: Case II - Pareto Efficient Frontier.

5.1.3 Case III

In this case the hydroelectric company's decision to allocate its PG is subject to the GSF that happened in 2015 (completely opposite from 2018) as seen in Fig. 5.8. This case has been made to measure of how robust is our decision from Case II.

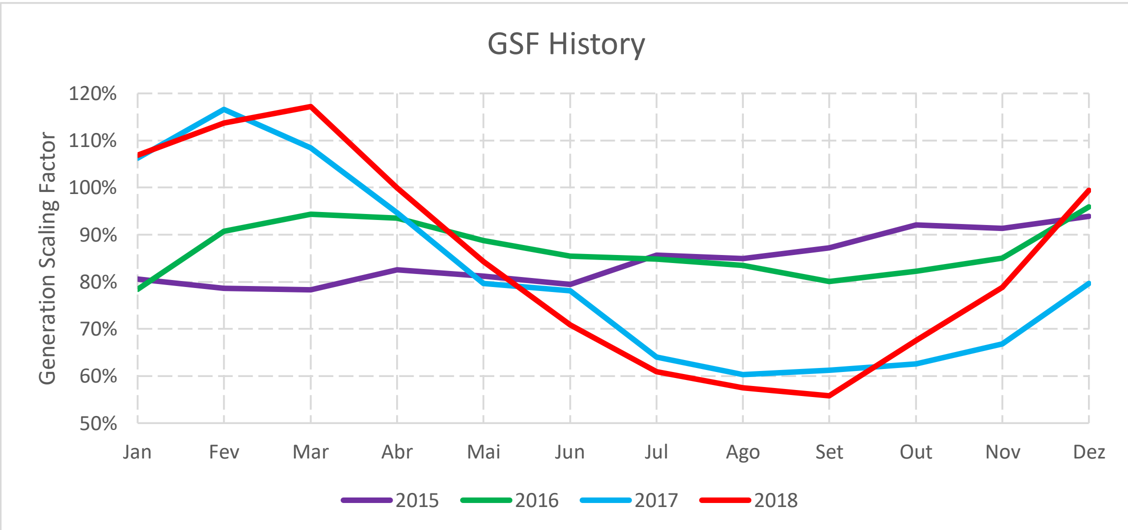


Figure 5.8: GSF history

In the risk neutral approach (illustrated in Fig. 5.9), the decision concerning the forwards contract is the same from the cases above. The decision to seasonalize the PG is to allocate part in the second semester and part in April, and May.

The risk averse approach (illustrated in Fig. 5.10) decides to buy forward contracts in the first semester, while selling a single contract in January. The PG allocation is divided along the first and second semester, with zero MWavg in January, July and August.

The difference from Case II is that, since the GSF in 2015 was higher in the second semester, the PG allocation is mostly located at the end of the year. The GSF is indeed an important parameter to estimate, since it can shift the seasonalization process.

Comparing the Return x Risk from Case II to Case III (illustrated in Figs. 5.7 and 5.11), the range of expected return doesn't change significantly. However, the minimum revenue in the latter is smaller, in the order of 10M€. This difference is explained by the fact that high prices (above *forwards* contracts) are obtained all over the year for the spot prices' projection, resulting on a risk neutral solution buying the available *forwards* and not selling any *forwards*. In this case, the effective risk is the occurrence of low spot prices at the end of each month, which are more likely to happen in the first trimester, especially in January. Looking at the GSF from Case II, that is significantly higher in the first trimester, the conclusion is that the risk is less intense than considering the GSF from case III, which does not suffer high fluctuations. The expected revenue, on the other hand, does not experience the same variation, since the average of the product $E[S_\pi]_m \times GSF_m$ does not change significantly along the year.

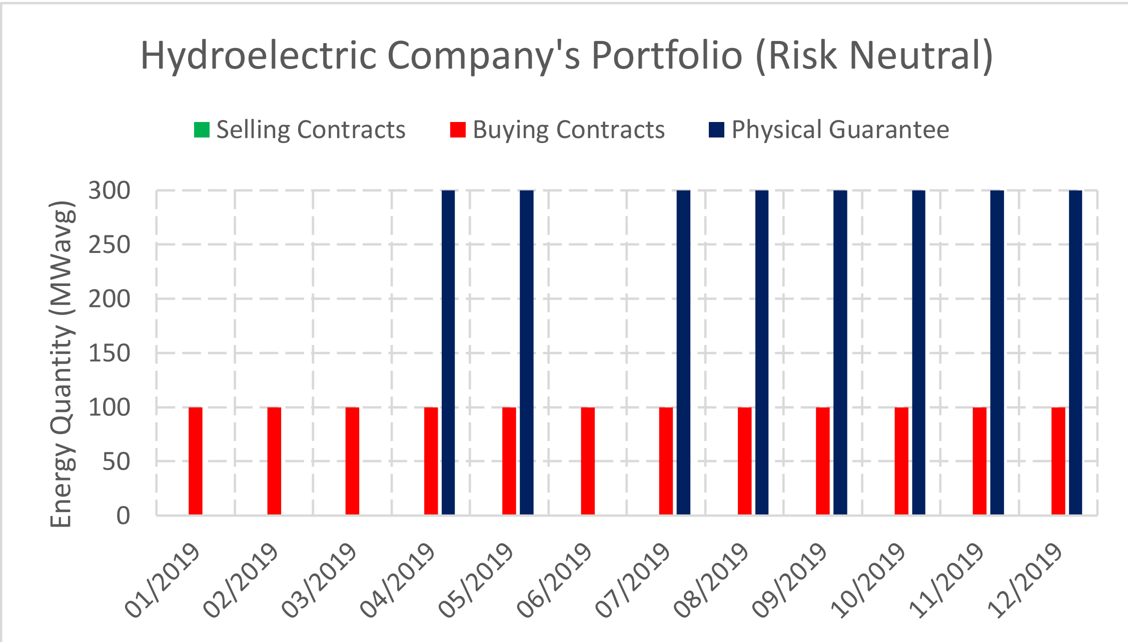


Figure 5.9: Case III - Risk Neutral Portfolio.

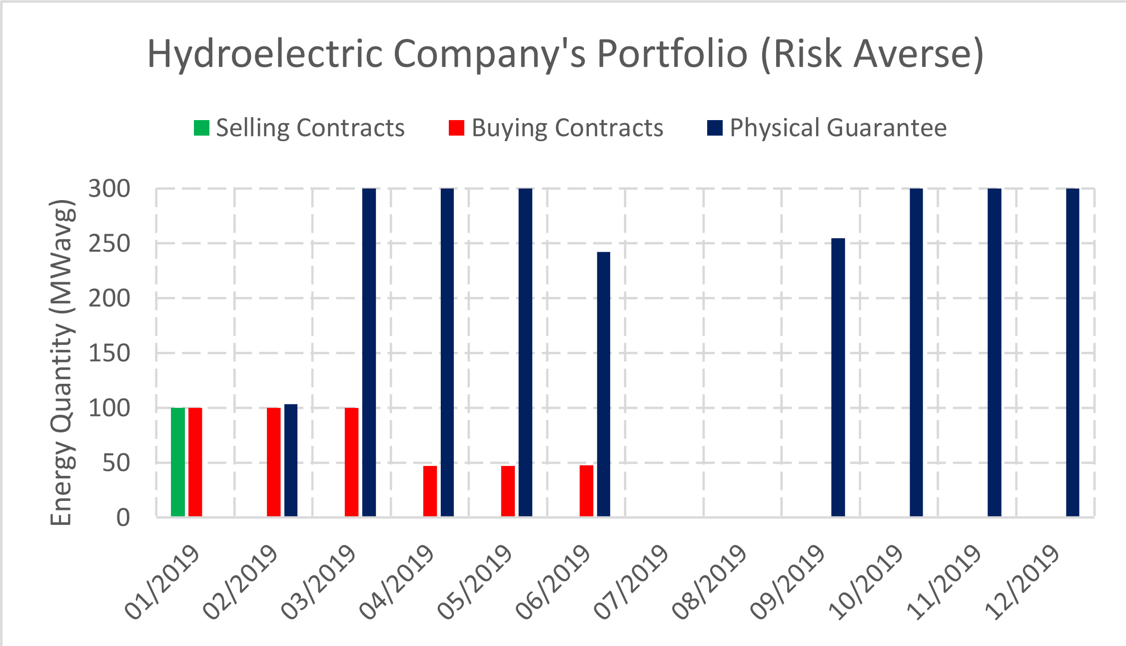


Figure 5.10: Case III - Risk Averse Portfolio.

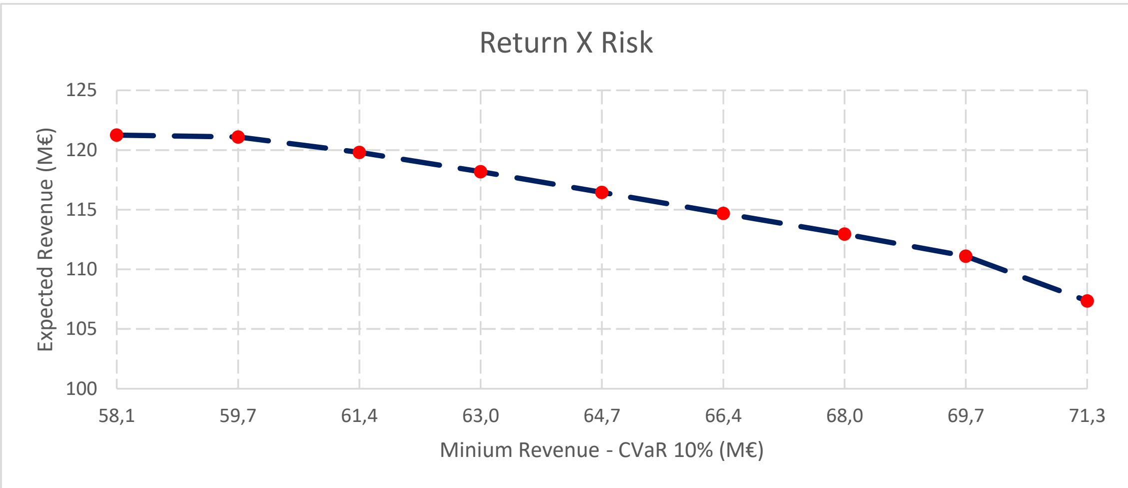


Figure 5.11: Case III - Pareto Efficient Frontier.

5.2 Scenarios Size Comparison

In this section the difference between two different approaches of realizations is compared, both cases take into account the GSF of 2018, and the possibility to seasonalize. The first one is as mentioned on the previous sections, to consider three spot prices (π_S) possibilities (High, Medium and Low) for January, February, Mars, Second Trimester, and the Second Semester, and the total combination of them, totalizing $3^5 = 243$ scenarios, that will be called *Simplified Scenarios Case*, this is illustrated in Fig. 5.12. In the second case three scenarios each month are considered, and the total combination of them totalize $3^{12} = 531.441$ scenarios for the whole year, that will be called *All Scenarios Case*. The risk metrics utilized in this section, is the CVaR with $\beta = 1\%$.

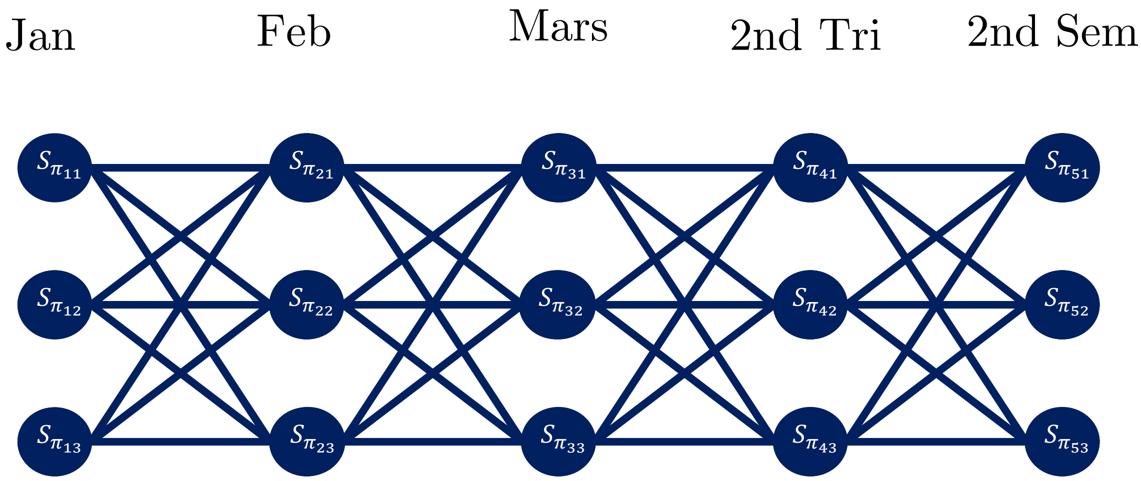


Figure 5.12: Scenarios Tree - Simplified Scenarios Case

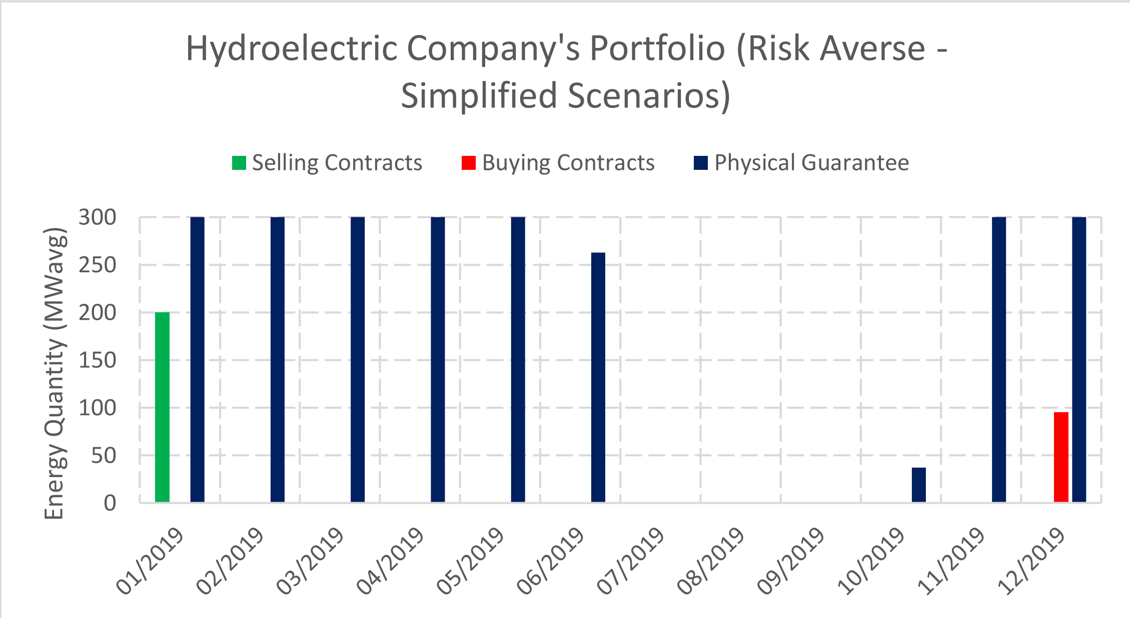
The results are illustrated first in Fig. 5.13a, where the optimal portfolio of the *simplified scenarios case* is plotted, then in Fig. 5.13b the optimal portfolio of *all scenarios case* is plotted. First of all, it is noticed that when a extreme of risk aversion is chosen, there are some similarities among them, i.e., the existence of a selling contract in January, as well as, the allocation of PG in January, even if the quantities varies. Further analysis shows that the PG allocation in the first semester remains almost identical, however, the buying contracts distribution along the year is different from one case to another.

In order to compare the Return vs. Risk for both cases, in Fig. 5.14a it is plotted the Pareto efficient frontier of the *simplified scenarios*, and in Fig. 5.14b it is shown the Pareto efficient frontier of *all scenarios case*. Analyzing these frontiers, it is noticed that the amplitude of the return and risk is reduced from the simplified scenarios case. It is effectively the consequence of taking into account the average of (approximately) 5300 critical scenarios (1% of 3^{12}), as opposed to 3 scenarios (1% of 3^5) for the simplified case. The average of 5300 scenarios will be higher, and give us less margin to operate the amplitude of risk.

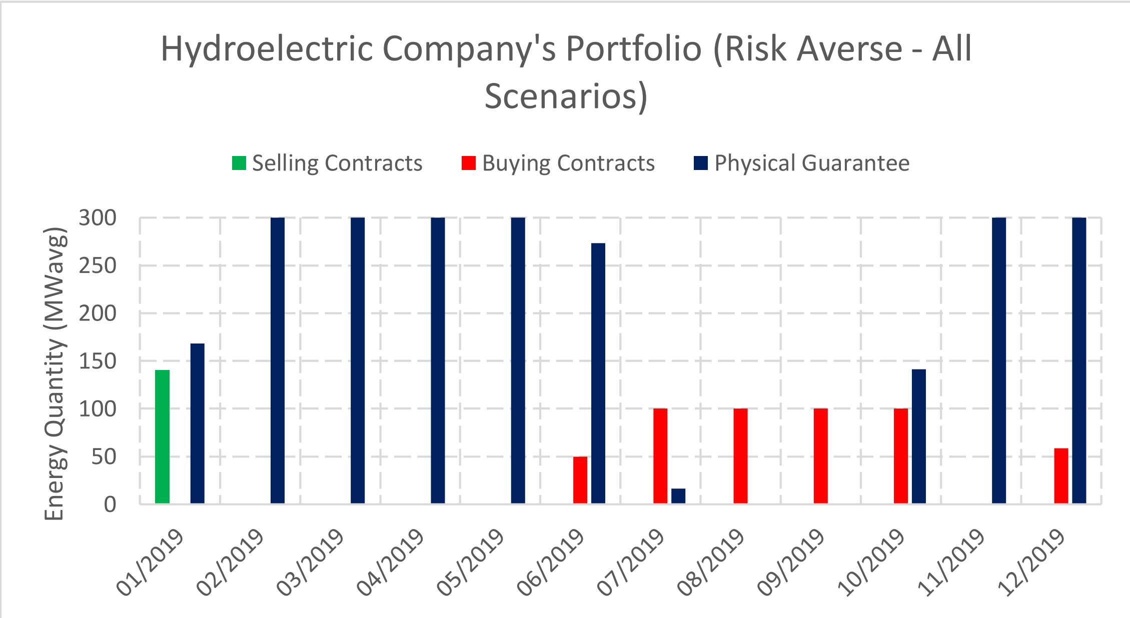
Therefore the decision agent should keep in mind that the quantity of scenarios has an significantly influence over the β CVaR. A word of precaution is that β should grow inversely proportionally to the number of scenarios.

Finally an important remark is that even considering the *all scenarios case*, the linear optimization could be done in a reasonable amount of time, which confirms that the linear optimization is well suited even for large problems.

A final remark concerning the *all scenarios case* is that in practice, it is seldom utilized due to the lack of good price projections for all the months along the year, instead it is more usual, to group the months by quarters, after the analysis of up to three months ahead, since as further in time the projections are made, the uncertainty increases sharply.

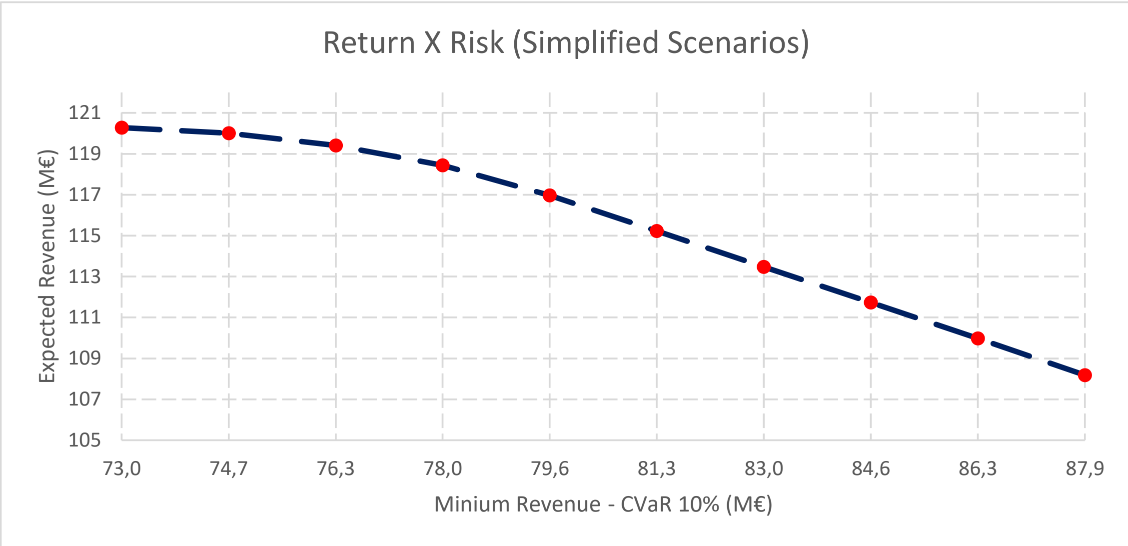


(a) Portfolio of Simplified Scenarios

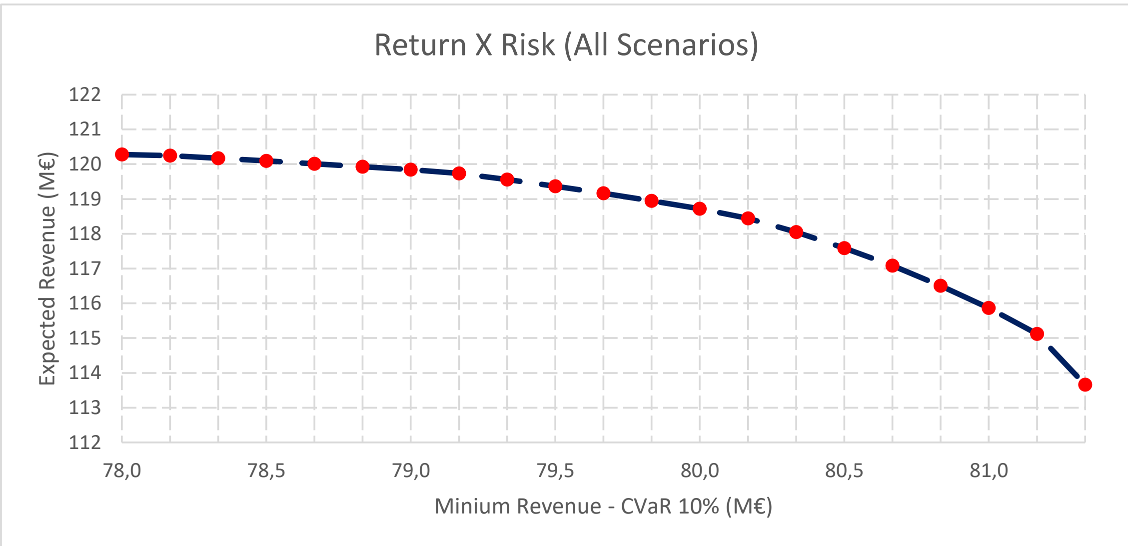


(b) Portfolio of All Scenarios

Figure 5.13: Portfolio Comparison



(a) Pareto of Simplified Scenarios



(b) Pareto of All Scenarios

Figure 5.14: Pareto Frontier Comparison

Chapter 6

Conclusions, Contributions and Future Work

6.1 Conclusions

This work presented a formulation of the problem of maximizing the revenue of a hydroelectric company in the Brazilian energy market, with CVaR constraints as a linear optimization problem, which proved to solve large scenarios case (over 500,000) in low computational time. Furthermore a tool for helping the decision maker of the PG seasonalization is presented. It has been shown that a GSF sensitivity scenario can shift the allocation of PG as the decision of other players varies, proving that the GSF is a crucial factor to estimate. It is worthy to notice the importance of spot prices projections, since the accuracy of these projections will directly influence the results. For active traders and generation companies, the proposed tool can be used every time when the view of the future changes. It is a dynamic tool for helping the energy market become a more strategic and competitive place. Finally, the proposed tool can be easily adapted to other market realities. A final important remark is that the data used on this study comes from a real life problem with some adjustments, furthermore it is useful to know that a hydroelectric company is using this tool to help achieve the optimal planning and operational decisions.

6.2 Contributions

The main contributions of this work, at the portfolio optimization of a hydroelectric company, are enumerated as follows:

- The development of a whole integrate tool to solve simultaneously the contracts optimization problem along with the PG allocation.

- A new approach (linear optimization) to solve the problem taking advantage of its formulation.

The results and contributions of this thesis achieved the publication of the following conference paper:

- RAGUENET, E., DIENE, O. "Decision support system for a portfolio of a hydroelectric company in the Brazilian market". In: 2019 16th International Conference on the European Energy Market (EEM), pp. 1-6. IEEE, 2019.

6.3 Future Work

Concerning the future work, it is proposed:

- The inclusion of different financial instruments in the portfolio, such as, derivatives contracts, e.g., put options, call options, collar contracts, and other hedging strategies;
- The inclusion into the modeling of the GSF projection as a strategic game, since it is affected by the decisions of other players, which can me accomplished by making use of the Game Theory approach.

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Appendix A

ERM Details and Algorithm

A.1 Introduction

This section intends to explain and exemplify the calculations of the ERM. In order to simplify the procedure of the ERM algorithm, without loss of generality it is assumed some hypothesis:

- The existence of only one submarket;
- The Optimization Energy Tariff (OET) from Itaipu is equal the ERM OET;
- There are only two hydro power-plants operating in the market (our hydro plant, hydroelectric A, versus a hydroelectric representing the market, hydroelectric B);
- The market hydro physical guarantee is much higher than our hydro plant physical guarantee($PG_A \gg PG_B$).

In order to develop all the calculations of the mechanism, it will be illustrated by four different scenarios that may occur in this situation. The market will be called the hydroelectric A, and our plant will be called hydroelectric B. For even further details of the mechanism we suggest reading [36].

In the following examples the physical guarantee of the market equivalent hydroelectric (PG_A) represents approximately 99.5% of all the PG of the ERM, while the physical guarantee of our hydroelectric (PG_B) represents approximately 5% off all the PG of the ERM.

A.1.1 Example 1

In this case it is considered the following conditions:

- $PG_A = 1000MWh$;$HG_A = 1100MWh$;

- $PG_B = 5MWh$; $HG_B = 6MWh$,

where the PG stands for the physical guarantee of each power plant, and HG stands for hydroelectric generation for each one of the power-plants.

In this scenario there is an excess of energy ($HG_A + HG_B \geq PG_A + PG_B$), therefore there will be *secondary energy*.

First as there is a surplus of energy, there is no need for readjustment of the PG. Then there is no necessity to cover the PG of any of the hydroelectric, since both generated above its PG. There is an excess of $100MWh$ and $1MWh$ respectively (difference between generation and the physical guarantee), so the total excess of the submarket is $101MWh$. Each hydroelectric has a right to receive an amount of the *secondary energy* proportionally to its PG. So the distribution of this *secondary energy* will be as follows: $SEC_A = 101 \times \frac{1000}{1005} = 100.5MWh$, and $SEC_B = 101 \times \frac{5}{1005} = 0.5MWh$, respectively. That means HE_A will receive $0.5MWh$ from HE_B. This will be valuated at the EOT.

The calculations concerning the amount of contractual energy that will be in plant B, is derived:

$$E_B = PG_B + \frac{PG_B}{PG_A + PG_B} \times ((HG_A - PG_A) + (HG_B - PG_B)) \tag{A.1}$$

$$E_B = PG_B + \frac{PG_B}{PG_A + PG_B} \times ((HG_A + HG_B) - (PG_A + PG_B)) \tag{A.2}$$

$$E_B = PG_B + PG_B \times ((GSF - 1) \tag{A.3}$$

$$E_B = PG_B \times GSF \tag{A.4}$$

It has been proved that the total contractual energy allocated in plant B will be $E_B = PG_B \times GSF$.

A.1.2 Example 2

In this case it is considered the following conditions:

- $PG_A = 1000MWh$;$HG_A = 1100MWh$;
- $PG_B = 5MWh$; $HG_B = 4MWh$

In this scenario there is an excess of energy ($HG_A + HG_B \geq PG_A + PG_b$), therefore there will be *secondary energy*. First there is no need for readjustment of the PG. Then there is the necessity to cover the PG of hydroelectric B, since its generation did not match its PG by $1MWh$. There is an excess of $100MWh$ from HE_A, and a deficit of $1MWh$ from HE_B to cover, so the total excess is $99MWh$, and each hydroelectric has a right to receive an amount of the *secondary energy* proportionally to its PG. Consequently the distribution of this *secondary energy* will be as follows: $SEC_A = 99 \times \frac{1000}{1005} = 98.5MWh$, and $SEC_B = 99 \times \frac{5}{1005} = 0.5MWh$. That means that HE_B will receive $1.0MWh$ from hydroelectric A to cover its PG, and additionally $0.5MWh$ from the excess of *Secondary Energy*. This ERM flux will be valuated at the EOT.

The demonstration of total contractual energy of HE_B from example 1, still holds in that case, i.e., $E_B = PG_B \times GSF$.

A.1.3 Example 3

In this case it is considered the following conditions:

- $PG_A = 1000MWh; HG_A = 900MWh$;
- $PG_B = 5MWh$; $HG_B = 6MWh$

In this scenario there is not an excess of energy ($HG_A + HG_B \leq PG_A + PG_B$), but rather of lack of energy, therefore there wont be *secondary energy*.

First there is a need for readjustment of the PG, because there is the necessity to cover the PG of hydroelectric A, however, it will not be possible to cover it entirely. Following the rules to adjust the PG, the new values will be $PG_A^* = 1000 \times \frac{906}{1005} = 901.5MWh$ and $PG_B^* = 5 \times \frac{906}{1005} = 4.5MWh$, respectively. There is an excess of $1.5MWh$ of generation from HE_B (difference between the generation and its adjusted PG), so it will be used to cover (part of) the deficit from HE_A. This will be valuated at the EOT.

The mathematical formulation to the new contractual energy of hydroelectric B is straightforward in this case, given by:

$$E_B = PG_B \times GSF \tag{A.5}$$

The final amount of each plant will still be $PG_B \times GSF$.

A.1.4 Example 4

In this case it is considered the following conditions:

- $PG_A = 1000MWh; HG_A = 900MWh$;
- $PG_B = 5MWh$; $HG_B = 4MWh$

In this scenario there is not an excess of energy ($HG_A + HG_B \leq PG_A + PG_B$), but rather a lack of energy, therefore there wont be *secondary energy.*

First there is a need for readjustment of the PG. Then there is the necessity to cover the PG of both hydro, however, as the $GSF \leq 1$, first the PG adjustment must be calculated to notice, which deficit should be covered. Recalculating the PG, we obtain the following values: $PG_A^* = 1000 \times \frac{904}{1005} = 899.5MWh$ and $PG_B^* = 5 \times \frac{904}{1005} = 4.5MWh$. There is an excess of $0.5MWh$ of generation from HE_A (difference of its generation from its adjusted PG),so it will be used to cover the deficit from HE_B. This will be valuated at the EOT.

The final amount of each our hydro will still be $E_B = PG_B \times GSF$.

A.1.5 Conclusion

We conclude this section by restating that, after analyzing and exemplifying all cases that may occur in the ERM, the amount of contractual energy in all cases will be $E_B = PG \times GSF$. A word of caution is that the agents in the ERM will buy/sell the ERM energy flux, at the EOT, which is a low value compared to the spot prices of 2018 (EOT_{2019} =R$12.41 $\approx$ 3€). In order to deal with the OEM valuation of this energy difference transaction, it would be necessary to include generation projections from both market, and our hydroelectric, then calculate the ERM algorithm, at each time period, to determine, which one should be receiving or giving energy. Since there will be fluctuations from both receiving, and giving energy at a very small tariff, at each time period, in this work it has been decided to not consider this EOT valuation, and keep only the final amount of energy to each hydroelectric.