MULTI-AREA RELIABILITY ASSESSMENT WITH VARIABLE ENERGY RESOURCES AND OPTIMAL IMPORTANCE SAMPLING BASED ON MARKOV CHAINS MONTE CARLO

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Dissertação de Mestrado apresentada ao Programa de Pós-graduação em Engenharia Elétrica, COPPE, da Universidade Federal do Rio de Janeiro, como parte dos requisitos necessários à obtenção do título de Mestre em Engenharia Elétrica.

Orientadora: Carmen Tancredo Borges

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AVALIAÇÃO DA CONFIABILIDADE MULTI-ÁREA COM FONTES DE GERAÇÃO RENOVÁVEIS E AMOSTRAGEM ÓTIMA POR IMPORTÂNCIA COM MONTE CARLO VIA CADEIA DE MARKOV

Daniela Bayma de Almeida

Março/2020

Orientadores: Carmen Lúcia Tancredo Borges

Programa: Engenharia Elétrica

A crescente penetração de fontes variáveis de energia ao redor do mundo trouxe alguns desafios para o planejamento dos sistemas elétricos. A análise da confiabilidade tornou-se mais complexa devido à necessidade de representar a variabilidade e correlação espacial entre as fontes (efeito portfolio) além dos diferentes perfis de geração em menor escala de tempo. Comumente utiliza-se a Simulação Monte Carlo (MC) para a análise da confiabilidade. Entretanto, o número de amostras necessárias é diretamente proporcional à variância do estimador da função de avaliação. Como os sistemas elétricos são confiáveis, a variância é grande e muitas amostras são necessárias para se determinar os índices de confiabilidade dos sistemas.

Desta forma, este trabalho tem por objetivo propor uma metodologia de Amostragem por Importância (IS), uma técnica de redução de variância, para análise da confiabilidade de sistemas em multiárea, reduzindo o tempo de simulação por Monte Carlo. Este trabalho considera uma representação *detalhada* de fontes variáveis de energia, considerando três técnicas principais: (i) Cadeias de Markov de Monte Carlo para obtenção de cenários de corte de carga; (ii) Estratificação para os perfis diários de geração renovável; (iii) Cálculos analíticos para definir limites para a LOLP e desenvolver fatores de ponderação ideais para a amostragem por MC.

A metodologia é ilustrada com estudos de caso de sistemas reais, onde os cálculos eram duas ordens de magnitude mais rápidos que o MC padrão.

Abstract of Dissertation presented to COPPE/UFRJ as a partial fulfillment of the requirements for the degree of Master of Science (M.Sc.)

MULTI-AREA RELIABILITY ASSESSMENT WITH VARIABLE ENERGY RESOURCES AND OPTIMAL IMPORTANCE SAMPLING BASED ON MARKOV CHAINS MONTE CARLO

Daniela Bayma de Almeida

March/2020

Advisors: Carmen Lucía Tancredo Borges

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The fast insertion of variable energy resources (VER) worldwide has brought some challenges to the planning of electrical systems. The reliability analysis has become more complex due to the need to represent the variability and spatial correlation between the sources (portfolio effect) in addition to the different generation profiles in a shorter time scale. Monte Carlo Simulation (MC) is commonly used for reliability analysis. However, the number of samples required is directly proportional to the variance of the estimator of the evaluation function. Since electrical systems are reliable, the variance is large, and many samples are needed to determine the reliability indexes of the systems.

This work has the main objective to propose a methodology that applies Importance Sampling (IS), a variance reduction technique, to analyze the reliability of multi-area systems, reducing the Monte Carlo simulation time. It considers a *detailed* representation of VER, considering three main techniques: (i) Markov chains Monte Carlo to obtain load shedding scenarios; (ii) Stratification for the daily profiles of renewable generation; (iii) Analytical calculations to define limits for the LOLP value and develop ideal weighting factors for MC sampling.

The proposed methodology is illustrated with case studies of real systems, where the calculations were two orders of magnitude faster than the standard MC.

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1 Introduction

1.1 Importance of supply reliability

A critical concern of planners and regulatory agencies in the design of electricity markets is to ensure a *reliable* energy supply in every moment. The chaotic situation during the 2003 blackout in New York City illustrates the importance of reliability: people were trapped in subway cars and elevators; gas stations stopped working because pumps rely on electricity; because credit card and cash-withdrawal machines were also out, one could not purchase supplies; staying in hotels was also impossible because the room doors used keycards. Therefore, the only option for several thousand people was to walk across the bridges to go home. Luckily, the blackout atypically occurred during warm weather; if it had taken place during a winter storm, the consequences would have been tragic. Since then, the economic and social impact of a power outage has increased even more, due for example to the widespread reliance on internet and mobile services.

An obvious way to ensure reliability is to invest in additional infrastructure, for example building redundancies into the transmission and distribution systems and increasing generation reserve capacity. However, these investments lead to tariff increases, which affect consumer welfare directly (a larger share of income must be used to pay electricity bills) and indirectly (increased costs of products and services which have a significant electricity component). Therefore, it becomes necessary to find the best *tradeoff* between costs and reliability

1.2 Representation of supply reliability in system planning and market design

One possible approach to finding the best cost × reliability tradeoff is to estimate a unit cost (or cost curve) for the load supply interruption[1] and then minimize the sum of {investment + operation} costs and supply reliability costs, as shown in Figure 1.

[1] The interruption cost is obtained from econometric models and/or customer surveys. A typical international cost is US$ 10 thousand / MWh.

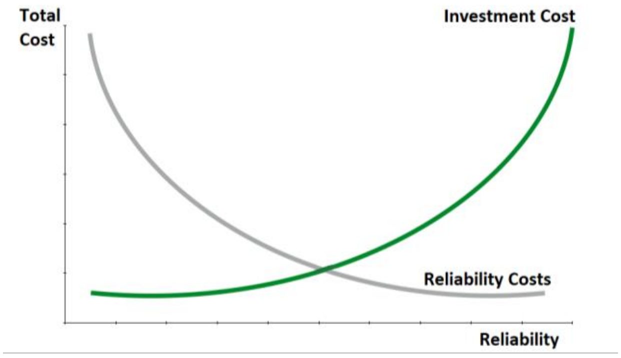


Figure 1: Trade-off between costs and investment to improve reliability

An alternative approach is for the planner (or market regulator) to establish a *supply reliability standard*. In this case, the objective of planning (or market design) is to minimize investment + operating costs, while ensuring compliance with the reliability standard.

The above objectives are shown in Figure 2. It is interesting to observe that the separate investment, operation, and reliability modules in the diagram closely match the subproblems of the Benders decomposition scheme, an iterative algorithm that is widely used for optimal system planning.

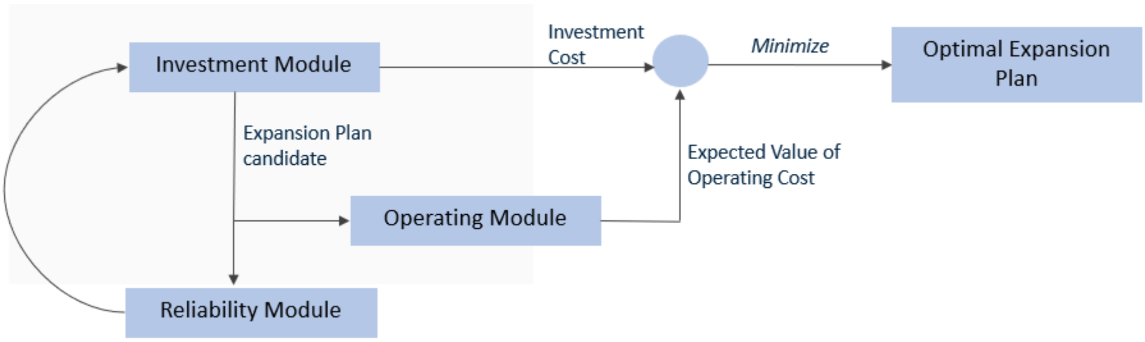


Figure 2: Energy Planning Flow Chart

1.3 Supply reliability indices

The supply reliability module, which is the focus of this work, estimates the *probability* of a load supply failure due to the combination of generation and/or transmission outages and load variation (the modeling of renewables will be discussed in the next section). The most common probability-related indices are $LOLP$ (loss of load probability) and $LOLE$ (loss of load expectation).[2] Those indices are estimated with basis on: (i) the probability of failure of each generator/transmission component; and (ii) the

[2] $LOLE = LOLP \times 8760$, i.e. the probability of failure expressed in hours per year.

execution of a *supply evaluation model* that determines whether each given *scenario* (combination of failures and load) results in a supply failure.

The supply evaluation model will be discussed in detail in later sections of this work. In a simplified way, the model determines the *minimum amount of load curtailment* required to eliminate the violations of system operating constraints such as generation and load balance and circuit overloads. This means that, in addition to estimating the probability of supply failures (LOLP), the supply reliability module can estimate their *severity*. The most common severity-related indices are $EENS$ (expected energy not supplied) and $EPNS$ (expected power not supplied).[3]

Historically, $LOLP$ was used as a supply reliability standard, e.g. the well-known "one day in ten years" criterion: $LOLP \leq \frac{1}{8760 \times 10} \approx 1.1 \times 10^{-5}$ [1] Later, severity-based standards were introduced, e.g. "$EENS \leq 0.2\%$ of average load". More recently, reliability standards based on the *conditional value at risk* (CVaR) were proposed [2][3], such as: "expected unserved energy conditioned to the 99%-100% quantile of supply severity $\leq$ 5% of average load".[4]

Historically, modelers took advantage of the fact that equipment outages were independent random variables and that supply failures were more likely to occur during the peak hours to develop efficient supply reliability algorithms, for example by convolving the generators' unit outage probabilities to obtain the probability distribution of total available capacity and by concentrating the reliability evaluation on the peak load periods (stratification). However, the very fast penetration of Variable Energy Resources (VER) resources such as wind and PV solar has created new modeling challenges for supply reliability evaluation. This topic will be discussed next.

1.4 The Variable Renewable Energy revolution

The original driver for Variable Renewable Energy (VRE) development was the concern about limiting global temperature increase due to greenhouse gas (GHG) emissions. Worldwide, most GHG emissions are caused by generation from coal plants. The concern about coal-based generation was compounded by the fact that they are main expansion option of fast-growing and very large countries such as India and China. Therefore, it became crucial to develop "clean" (i.e. non-emission) and inexpensive

[3] $EENS = EPNS \times 8760$, i.e. the average MW not supplied expressed in terms of MWh per year.
[4] Other supply reliability measures used are LOLF (loss of load frequency) and LOLD (loss of load duration).

generation sources that could displace new coal capacity and, in a later stage, replace existing coal plants. Although several sources such as hydro, biomass, geothermal and nuclear qualify as clean, the focus was almost entirely on wind and PV solar. One attractive characteristic of wind and solar is that they are available practically everywhere, whereas competitive hydro, biomass and geothermal are concentrated in fewer countries. For various political reasons - concern about nuclear accidents such as Fukushima, use of plant fuel enrichment to produce nuclear weapons etc. - as well as higher cost and longer construction time, nuclear was excluded as an option in most countries.

The VER revolution was led by Germany, Denmark and other EU countries, plus the US West Coast. They created a set of incentives and subsidies that were very successful both in terms of installed capacity and unit cost.

The graphs shown in Figure 3 and Figure 4, taken from the International Renewable Energy Agency (IRENA) [4] illustrate the VER growth. The first graph shows the evolution of total renewable capacity (VER plus hydro, biomass etc. except nuclear) since 2000, whereas the second shows the capacity increase of each technology since 2010.

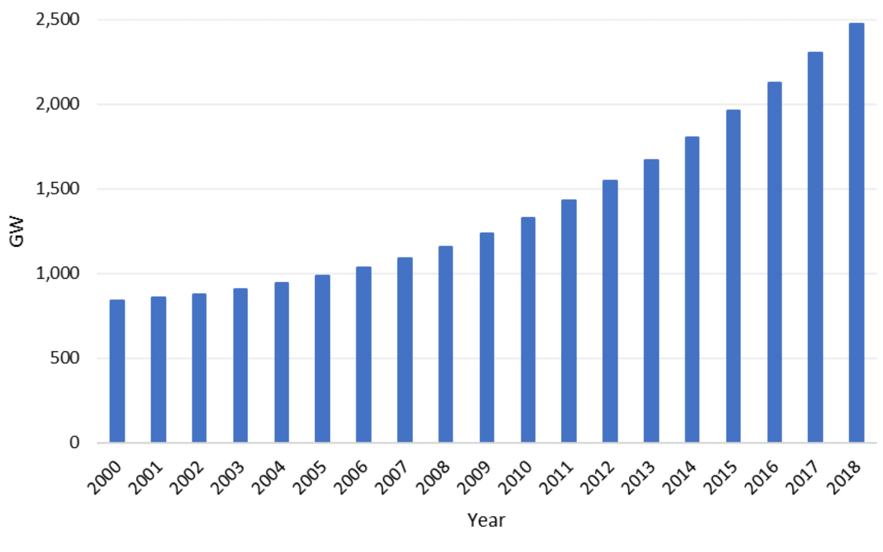


Figure 3: Total renewable energy in the world (GW)

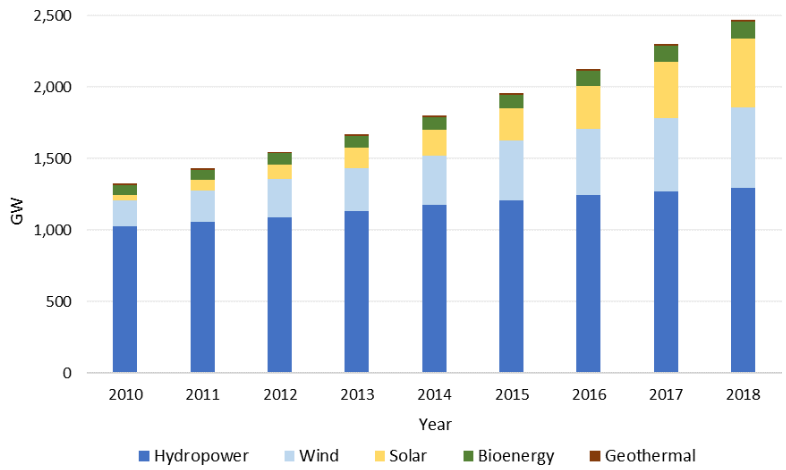


Figure 4: Total renewable energy in the world, per technology (GW)[5]

1.5 Supply reliability evaluation with Variable Energy Resources

The presence of Variable Energy Resources (VER) sources increases the complexity of supply reliability assessment in four aspects: (i) shape of net load; (ii) hourly resolution; (iii) modeling VER intermittency; and (iv) representation of regional interchange limits. Each of these aspects is discussed next.

1.5.1 Shape of net load

It is intuitive that supply failures are more likely to occur during periods where the difference between available capacity and load is smaller. Because thermal plant outages occur randomly along the day, those periods coincided with the peak load. Many supply reliability methods took advantage of this fact, for example by having more outage sampling for the peak hours (stratification).

However, this situation changes with VER, because their generation pattern changes along the day. Therefore, the periods with the highest net load (demand minus variable energy production) are likely to be different than those of peak load. This is illustrated by Figure 5 without solar generation, peak load occurs at 3 p.m. With solar, the peak net load moves to 7 p.m.

[5] Marine source has not been included in the Graph due to its total small capacity. Marine comprehends tide, wave and ocean technologies.

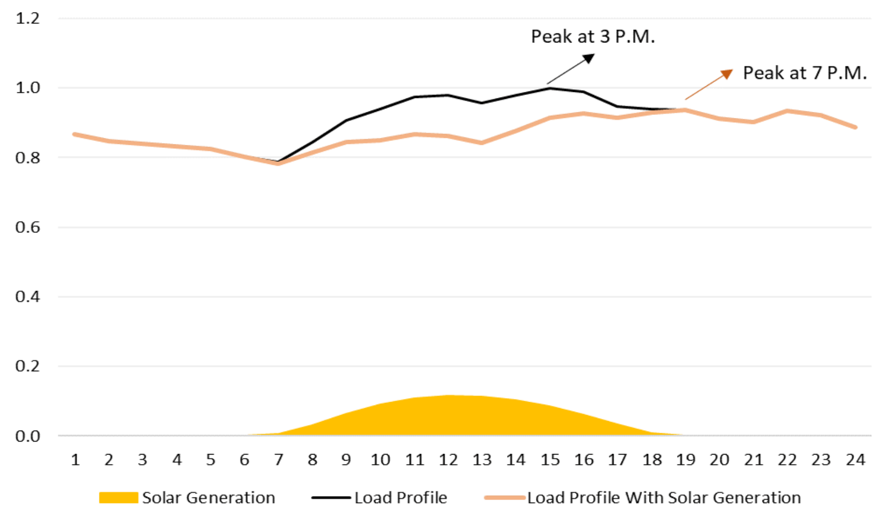
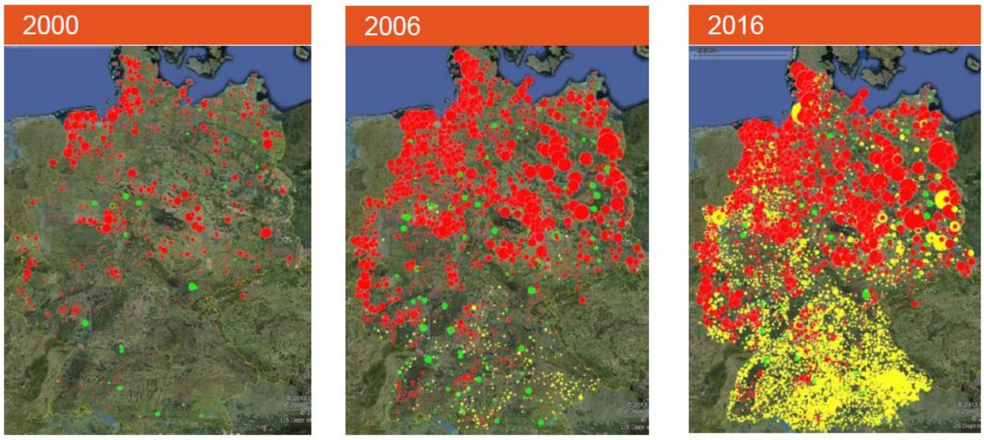


Figure 5: Peak load with and without VER sources

1.5.2 Time resolution

With VERs, it is no longer possible to use a "coarser" resolution, such as peak, intermediate, and off-peak "time blocks"; VER modeling requires at least hourly resolution to represent intermittency. Figure 6 illustrates the fast and massive insertion of renewables sources (mainly wind and solar) in Germany for the years 2000, 2006 and 2016. The dots in red represent wind sources, in yellow represents solar sources while the green ones indicate biomass.



Source: 50Hertz, TenneT, Amprion, TransnetBW, Google Earth

Figure 6: Renewable Sources in Germany over the years 2000, 2006 and 2016

1.5.3 VER time and spatial dependence

The third aspect that should be considered is the modeling of spatial and temporal correlation among VER sources. The correlation among sources impact on the planning and operation of the system once it influences on the share of power generation in the system. The importance of capturing spatial correlation between wind sources for adequacy analysis is highlighted in [5] and [6].

In the context of reliability adequacy assessment, a positive temporal correlation among variable sources may result in a higher availability generation capacity in the system contributing to the load supply. If this temporal correlation is not considered in reliability studies the reliability indexes could overestimates the reliability indexes, or in other words, sub estimate the system reliability.

Similarly, not considering an existing spatial correlation among intermittent sources could lead to a misleading reliability index since it would not represent the real available generation capacity to supply the system load.

Figure 7 illustrates the effect of spatial correlation in Chile between one hydro plant and two wind farms. The hydro and one wind sources are +0.55 correlated whereas the same hydro with a more southerly wind farm have a correlation of -0.6. Figure 7 indicates these sources location and the water inflow (in m^3/s) and wind generation (in p.u.) for the same month in 44 different years.

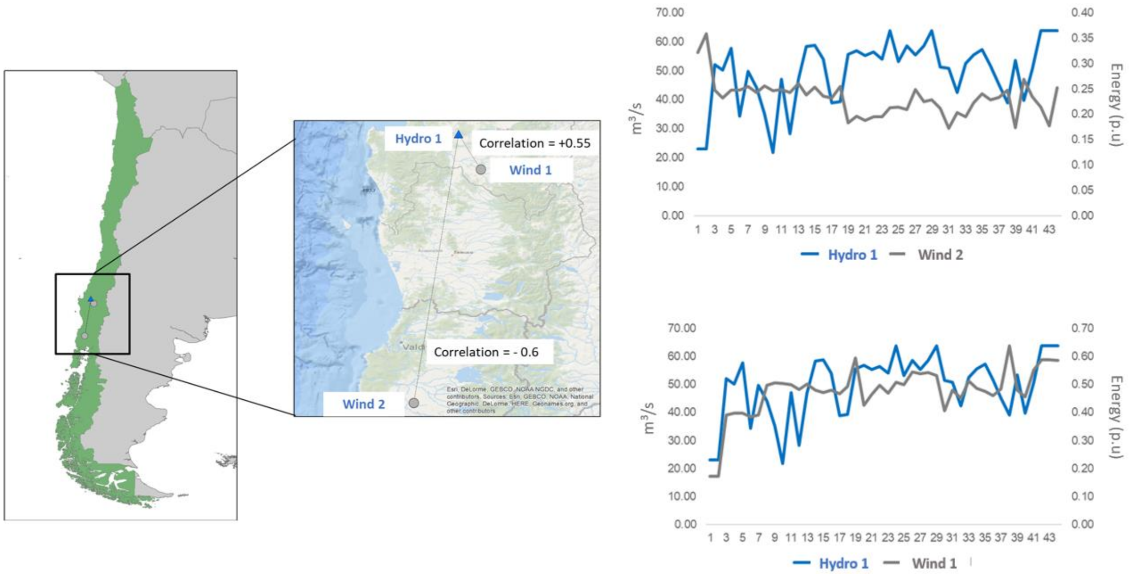


Figure 7: Spatial Correlation among hydro and wind sources in Chile.

1.5.4 Regional interchange limits

Interconnections have always been essential in electrical systems because they increase security, for example by sharing generation reserves, and reduce operating costs through exports to areas with higher marginal costs and vice-versa, imports from areas with lower marginal cost.

The growth of VER increased the importance of interconnections in many countries. One reason is the so-called “portfolio effect”, where VERs in different areas with complementary production profiles and/or low spatial correlation can be aggregated. Suppose, for example, that one area has wind power, which tends to be higher during the night, whereas another area has solar power, which obviously produces during the day. The interconnection of those areas allows a 24-hour “firm” energy production.

A second reason for the increased importance of interconnections is the fact that the best solar and wind sites may be in areas which are far from the main load centers and, thus, require the construction of new transmission links. Two examples are Chile, for solar in the Atacama Desert; and Texas, for wind.

“Decarbonization” policies are a third reason: interconnections allow countries to import surplus “cheap and clean” VRE from their neighbors, thus reducing their fossil fuel generation. For example, in 2014 the European Council set a 10% *interconnection target* (defined as the ratio between net transfer capacity and installed generation capacity) for the Member States, to be achieved by 2020 [7]. More recently, this target was increased to 15%, to be achieved by 2030. An Expert Group was created to provide advice to countries on how to achieve this target and to study any issues related to interconnection capacities. According to the benefit/cost report [8], for a 1.8 billion Euros network investment until 2030, the yearly operating cost reduction would be 40-70 billion euros.

In order to determine the tradeoff between the above-mentioned benefits and the costs of interconnections, expansion planning studies are carried out in two hierarchical steps. The first step carries out a co-optimization of generation investment plus expected operation costs and interconnection costs, using an aggregated model of the power grid, the so-called multiarea representation. For example, the Figure 7 below shows the seven areas used for renewable insertion studies in Brazil and the six-area system used for Chile.

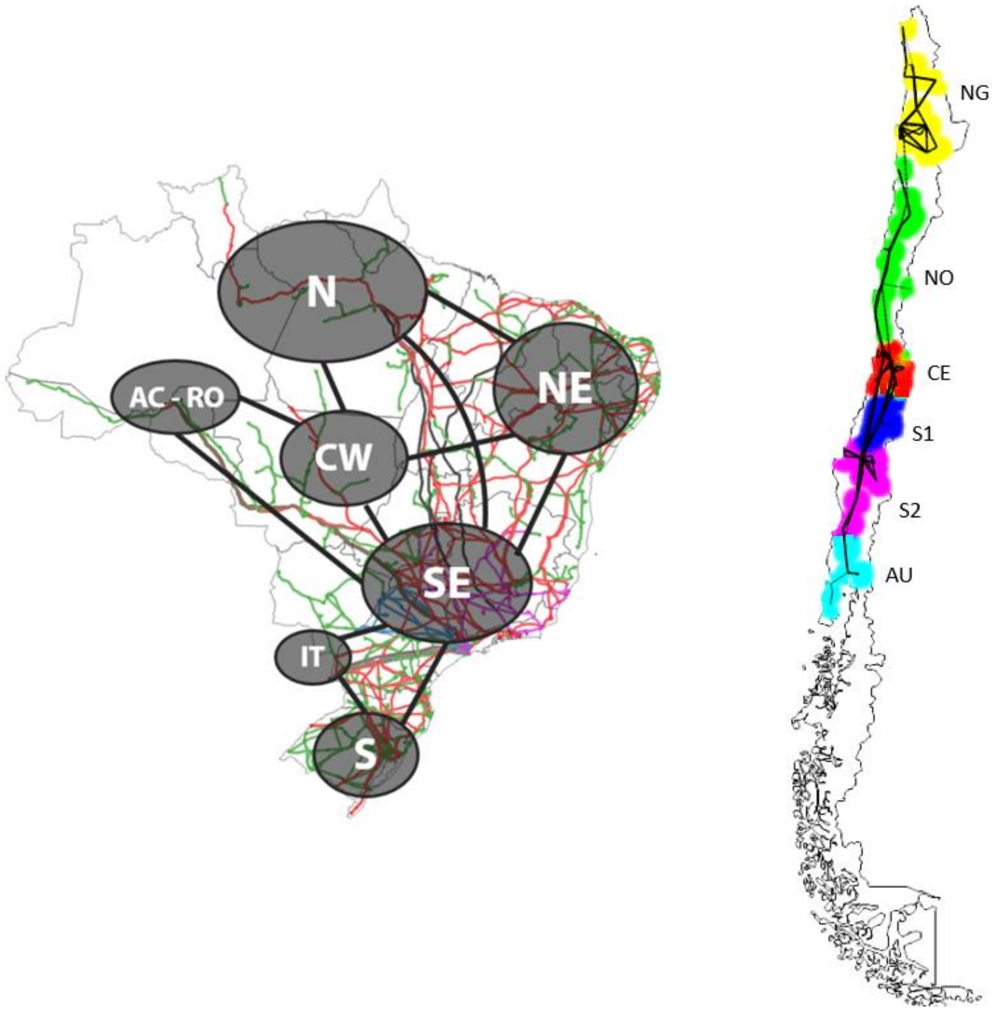


Figure 8: Seven areas in Brazil and six areas in the Chilean system.

This first co-optimization step captures the details and the synergy of the renewable generation profiles in neighboring regions. In the second phase, the electrical network is analyzed in detail in order to check if there are congestions in the grid or voltage problems. The second step may conduct to the necessity of transmission network expansion (transmission lines and transformers, for example) or reactive reinforcements (such as inductors and capacitor bancs).

1.6 Objectives and relevance of this work

As mentioned, the fast insertion of renewable sources worldwide introduced some challenges to the reliability evaluation such as the necessity to use stochastic models to produce renewable scenarios, consider correlation between renewable sources and use of hourly resolution to capture the variability of renewable sources and its effects in the system.

Thereby, this work will propose and describe methodologies for the power reliability assessment including a representation for VER, time and spatial correlation (portfolio effect) and daily profiles with hourly (or smaller) resolution considering multi-area systems. In other words, it’s considered that power electric systems are defined as areas interconnected with circuits, but not representing the detailed network.

For the sake of a power system assessment it’s proposed a methodology to optimize the Importance Sampling (IS), a well-known variance reduction for multi-area reliability assessment with VER based on the combination of three techniques: (i) sampling of failure states by Monte Carlo Markov Chain (MCMC); (ii) stratification of the daily profiles of VER production; and (iii) use of upper and lower bounds of LOLP for the strata to develop optimal weighting factors for the Monte Carlo (MC) sampling.

1.7 Organization of the dissertation

Table 1 shows the chapter organization of this dissertation.

Table 1: Chapter Organization

Chapter	Description
2	Review of reliability evaluation methodologies, with emphasis on multiarea reliability assessment.
3	Background Knowledge which includes a multi area representation for modelling a power system and an overview of the Monte Carlo Simulation (MCS) method
4	Description of Importance Sampling and Markov Chain Monte Carlo. These two methods are considered in the methodology proposed in this work.
5	Proposed Methodology for Multi Area reliability Assessment. It presents flow charts to better illustrate the techniques used and highlights the advantages of the proposed methods over traditional ones. The new mathematical formulation for determining the Optimum Importance Sampling (IS) based on MCMC is presented. It also contains the description of the Stratification Technique (ST) which is the proposed methodology for representing renewable sources in reliability studies.
6	Results of two case studies. The first one with a Saudi Arabia-derived system and the second case study considering the IS+MCMC+ST methodology to a Chile-derived system.
7	Conclusions and Future Works

2 Literature Review

2.1 Importance of probabilistic reliability assessment

Ideally, electrical power supply should be both 100% *reliable* - i.e. consumers are never interrupted or suffer power quality degradation (“brownouts”) - and *economic*, i.e. consumer tariffs should be reasonably low. However, those objectives are *conflicting*. On the one hand, generation and transmission equipment is subject to unexpected outages, caused by a wide range of factors: component malfunctions, weather conditions, (mis)actuation of protection devices, operational errors and others. In addition, energy production of hydro and VER, as well as demand, are intrinsically variable. Therefore, the only way to ensure full reliability is to have high redundancy of generation and transmission (ENDRENYI [9]) and plan the system to withstand the simultaneous occurrence of extreme scenarios, such as lowest VER production, most severe drought and highest load. Obviously, the above measures will result in unacceptably high tariffs. Therefore, it becomes necessary to determine an acceptable *tradeoff* between a certain amount of supply failures and consumer tariffs.

One possible way to achieve this tradeoff is to use *deterministic criteria*, for example, plan generation reserve to be 30% of peak load [11]; ensure load supply even if the worst drought observed in history occurs again; ditto if any circuit fails (N-1 criterion); and so on. These deterministic criteria are easy to understand and implement; however, they also have significant *limitations*, because they do not capture the *joint probability* of several factors, such as equipment outages and VER production. As a consequence, it is never clear whether the system is actually under-installed or over-installed, or even if the supply failures will be homogeneous, i.e. occur with the same frequency and severity for consumers in different nodes of the grid.

Due to these limitations, the assessment of supply reliability considering the probability of failures and other stochastic aspects such as load and VRE variation is seen as a more adequate approach. According to BILLINTON and ALLAN in [10], the first studies that analyzed generation system reliability considering stochastic approaches were published around 1930.

2.2 Hierarchical levels for reliability assessment

According to BILLINTON and ALLAN [10], reliability assessment methods can be classified to three hierarchical levels:

Hierarchical level 1 (HL1) represents only the outage of generating units; the transmission network is disregarded, i.e. represented as a single node. Therefore, HL1 provides information about generation reliability. Later, utilities discovered that sharing reserves with neighboring areas could improve and reliability. Thus, HL1 was extended to include multiarea reliability evaluation [11] which, as seen in Chapter 1, is the focus of this work.

In turn, Hierarchical level 2 (HL2) expands the analysis to consider the transmission network equations and constraints, as well as circuit failures. Including transmission in the reliability analysis increases the computational complexity because a system supply failure is no longer assessed by simply comparing the total available capacity with the total load, as in HL1; it is necessary to solve an optimal power flow.

Finally, hierarchical level 3 (HL3) also includes the distribution network equations, constraints and equipment failures, that is the entire system from generator to consumer. As can be imagined, computational complexity is further compounded for HL3 assessments.

2.3 Security and adequacy

In addition to the hierarchical levels, reliability studies are classified as *security* or *adequacy*. Security assessments include dynamic stability analysis, whereas adequacy refers to "steady state" evaluations. This work focuses only on adequacy assessment; therefore, from now on, the terms "reliability" and "adequacy" will be used as synonyms.

2.4 Methods for reliability evaluation

Supply reliability methodologies are usually divided into *analytical* and *simulation* methods.

2.4.1 Analytical methods

Analytical methods evaluate all possible system *states*, defined as a vector of generation/transmission equipment availability (typically, a binary index, working or failed), load level and VRE production and their respective probabilities. Due to the combinatorial increase of states with the number of system components, most methodologies use *implicit enumeration methods* such as the convolution of generation capacity distributions to obtain the probability distribution of total generation capacity. Other analytical methods include Contingency enumeration and Minimum Cut Set. Contingency methods assess reliability indexes considering some contingencies in the system which should be carefully defined to not compromise the accuracy of reliability assessment. According to VRANA and JOHANSSON [14], the Minimum Cut Set method is useful for analyzing reliability in specific areas in the system, because it assesses the indexes for specific load points using minimum cuts.

Due to computational limitations, for a long time the only widespread analytical method was convolution for generation reliability evaluation. In the 1980s, an analytical approach was proposed for multiarea reliability studies, considering generation and interconnection outages [15].

Considering areas for the reliability evaluation is a simpler method compared to the real generation and transmission reliability problem. Even though simpler, multi area evaluation still required solving an optimization problem, for every dispatch scenario, to determine the maximum flow and then, verify if the load was supplied. Nevertheless, the maximum flow-minimum cut theorem proposed by FORD and FULKERSON in [16] proved that the maximum flow (primal of the optimization problem) is equal to the minimum cut (dual problem). Cuts in a graph representation separate the source and sink nodes indicating there's impossible to supply the demand. Minimum cuts correspond to the minimum set of arcs that once removed prevent load supply. The sum of all arcs removed to form a cut defines the cut capacity. [18]

With this theorem, there's no need to solve an optimization problem for all scenarios. It's possible to determine all cuts in a system at once and the minimum cut of each scenario is assessed by the available power in each area and the transmission lines capacities. Since maximum low-minimum cut avoids solving optimization problems for every studied scenarios, it reduces the computation time for a multi-area power system reliability analysis.

More recently, population-based intelligence algorithms were proposed as analytical methods to evaluate a system reliability [19] These methods usually use optimization techniques and genetical algorithms to evaluate the system reliability. The main idea behind these methods consist on reducing the state space to improve convergence in a way to obtain a good approximation for calculating the reliability indexes. Each iteration or generation of a population-based intelligence considers more than one state at the same time. [20]

Some of population-based intelligence (PIS) methods used for power systems reliability analyzes are genetic algorithms, particle swarm optimization, ant colony system and artificial immune system as mentioned by WANG, L *at al.* [21] and GREEN, R *at al*. in [22]. PIS methods are similar to optimization problems since use a fitness function to represent the target solution. PIS methods are iterative methods that create populations based on the previous ones and, attributes weights to each individual according to their suitability in the fitness function. So, throughout the iterations, these methods go towards states that more contribute to indexes calculation to draw samples closer to the target solution, and then, are more likely to have a faster convergence compared to Monte Carlo simulation. These methods deal with problems of memory management and prevention of visiting the same state space more than one time. [23][24]

2.4.2 Simulation methods

The stochastic simulation method, also named Monte Carlo simulation is based on sampling states of each system's component according to their probability of occurrence, trying to capture the random behavior of the system. Together the components' states indicate an operating system state. It's common to consider in reliability studies the network components (generators, transmission lines and transformers, etc) following a binomial distribution. It means that there are two possible states: operating state (on) or under repair (off).

Monte Carlo Simulation analyzes the power system calculating expected values of the indexes, without visiting all states in the system.

It is based on generating pseudo-random numbers, following a uniform distribution, for each element in the grid. Then this random number is applied into the component accumulative density function to check it corresponding state. If the pseudo-number is bigger than the component forced outage rate (F.O.R) it's on operating state (on). So, this approach requires awareness of components' failure rates.

There are two types of Monte Carlo simulation: non-chronological and chronological. In a non-chronological Monte Carlo Simulation, states are sampled without considering time sequence. So, for example, in a system of two generators (G1 and G2) each one with one unit, the sampled state of G1 and G2 don't, necessarily, correspond to the same period. The non-sequential technique requires less computation effort and then can be used in cases that neither time sequence is relevant nor information regarding frequency and duration events (such as duration of shortage energy).

On the other hand, the sequential Monte Carlo Simulation considers drawing states consecutively through time and then, the state of each component corresponds to the same point in time. Therefore, a sequential simulation expects not only the components failure rates but also, their historical and chronological series indicating their trajectory.

Besides considering the stochastic behavior of power system and being suited to large-scale systems, Monte Carlo Simulation (MCS) methods require many simulations to accurately determine a system adequacy and then a high computational effort.

It can be shown that the number of samples required in the MCS to estimate a system unavailability is directly proportional to the estimator's variance and inversely proportional to the squared of the desired accuracy in an index determination [25]. As shown in [26] 10,000 samples are sufficient to estimate a system LOLP with an accuracy level of 30%. However, reducing it to 3% would require 10^6 samples.

According to the 1990 annual ERIS report, as published by Li, W. in [26] the average LOLP for hydroelectric generators is 3.34E-02 pu, for fossil fuels units is 5.90E-02 pu. Transmission line failures due to line related outages results in an average LOLP of 5.43E-04 pu (for each 100km of line) and transformers' average LOLP is 6.65E-03 pu due to sub-components failures. It is led to conclude that power systems are usually reliable, and it is unlikely to sample states resulting in load shedding and contributing to statistics reliability.

Due to high computational effort and low converge of Monte Carlo simulation some techniques were proposed in the literature to improve the reliability analysis for a system.

2.4.3 Variance reduction techniques

Variance reduction techniques (VRT) aim to reduce the variance of an estimator, once reducing variance also decreases the number of samples to estimate the reliability indices by Monte Carlo simulation, for the same accuracy level.

Variance reduction techniques have been identified as a good alternative for improving the convergence of the algorithm MCS applied to power systems. Some VRT proposed in the literature are stratified sampling, antithetic variates (AV), control variates, importance sampling and cross entropy. [19]

Stratified sampling is similar to the importance sampling technique and aims to divide the state space into subgroups called strata with nonoverlapping populations. It lets sampling from strata that more contribute to interest indexes and as consequence reduces the variance of the analyzed estimator. [27] [28]

Antithetic variate technique uses two estimators that are negative correlated to determine an unknown parameter. [19] To determine EENS in a generating reliability analysis, CHEN and MILI [28] suggests mean time to failure (MTTF) and mean time to repair (MTTR) as estimators in the AV technique since MTTF and MTTR are negatively correlated to energy availability and to EENS either. AV can be used for reducing computation effort during calculation of load buses or systems indexes. [29]

Control Variates attempts to reduce a variable's variance considering a correlated explicative variable whose variance and expected value are known. [29]. OLIVEIRA *et al.* [30] proposes applying control variates to evaluate composite reliability considering generation capacity indexes (resulted from an HL1 analysis) as control variable. In [30] it is also suggested to use transmission outages as control variable for composite analysis. BILLINTON and JONNAVITHULA applied control variates in [29] in a composite reliability analysis and concluded that it reduced the computation time and the number of samples in more than three times if compared to the Standard Monte Carlo Simulation.

Conforming TOMASSON in [15], the importance sampling is a variance reduction technique which aims to reduce the variance of the probability density function of network's components to increase the chances of sampling system failure scenarios and then, reduce the number samples in Monte Carlo simulation. However, it's quite challenging to implement it, since it is not known the best way to "deform" the components' density function. The cross-entropy (CE) algorithm suggested by RUBISTEIN in 1997 [31] was motivated to propose a new approach for estimating rare events probability. Cross Entropy is an importance sampling method for changing the original density function to obtain another one more likely to sample rare events. [32] Some CE applications have been proposed in power systems reliability around the 2010's including in studies in HL1 reliability and composite reliability (HL2) either.

According to the Tutorial on the Cross-Entropy Method [33], cross Entropy is an iterative algorithm that can be divided in two main parts. In the first one, the main goal is to determine the reference parameter of the distorted density function **f (., v)** that has the same family distribution function of the original one **(f (., u)).** Once the reference parameter v is determined, i.e., an optimal distortion for the original density function, the reliability's indexes can be determined. The second stage consists in using Monte Carlo simulation and sampling from the new distribution function to analyze the system reliability. Modifying the density function is just an easier way to sample states of interest. So, to avoid biased or an incorrect index estimation, an "adjust factor" must be included since the samples are taken from the new distribution instead of the original one. This adjust factor is called likelihood ratio.

In [33] it's also shown that the referenced parameter calculated by CE algorithm can be determined analytically if the distribution function of the random variables belongs to the exponential family.

In the first part of CE algorithm, many samples must be drawn to determine the reference parameter of the distorted distribution function (v) and it may take much time due to the rareness of failure events required to determine it (v).

In [34] the authors suggest a simpler and analytical approach to use cross entropy in HL1 and get the parameter of the distorted distribution (v) considering each generating unit j (v_j). The calculation of v_j depends on each generation unit F.O.R, the system LOLP and the $LOLP_j^+$ that is the system LOLP considering the unit j on the up state. To determine the LOLP, the simplified CE algorithm considers the peak load as constant, and the generating distribution is obtained from the discrete convolution of each generating unit distributed function.

Some months later, the authors of [34] extended in [35] the CE methods for composed reliability analysis. Then, the original distribution of generators and transmission lines are modified to enlarge the probability of sampling failure states. Then, in the second stage of the CE method a Monte Carlo Simulation considering electric network to determine the reliability indexes is used. Despite reducing computational effort if compared to the Standard Monte Carlo Simulation, it proposes distortion of the distribution function of each component in the system. However, as power systems are usually reliable and most of the components do not contribute to the system failure state, modifying the distribution function of all components is not efficient.

Consequently, in a concise way, the cross-entropy methodology tries to optimize the importance sampling changing the probability density function by giving weight to the components that are on off state more frequently in system load shedding state.

In article [36] the authors point out that besides improving Monte Carlo convergence and reducing the solution effort, CE techniques lose accuracy if used in big dimension systems with rare events probabilities due to multiplying small numbers to compute the likelihood ratio. To overcome this issue the authors suggest a three-step methodology to determine the indexes for power systems reliability analysis. The first step determines bottle-neck components, i.e., the ones that contribute most to the failure scenarios and, therefore, only these will be distorted by cross entropy in the second stage. Bottleneck components are iteratively determined, and the convergence occurs if bottleneck components repeat in two consecutive iterations. In the third stage the indexes are then obtained. It is also mentioned that just few elements contribute to failure states and then, in few iterations, the algorithm converges and determines the bottle-neck components preventing degeneration issue.

Besides VRT and IS techniques to speed up Monte Carlo Simulation, some papers suggest using subset simulation as technique to sample rare failures events. Opposite to IS techniques that aims to alter a probability density function, subset simulation assesses unlike regions on the space by replacing rare failures events by a sequence of simulations of more frequent events using conditional probability. So, it’s possible to determine a failure event probability multiplying conditional probabilities of intermediate states. It’s a technique that aims to reduce the state space to a subset of interest. [37]

Subset simulation is a Markov Chain Monte Carlo (MCMC) algorithm. MCMC’s first proposed algorithm is *The Metropolis* published by METROPOLIS in 1953 [38]. Its main idea is moving towards some interested area in a very complex probability density function in a multi-dimensional space. However, this algorithm was not very profitable for high dimensional spaces since samples generated from Markov chain had high correlation and then would lead to biased estimators.

AU, S. K. and BECK, J. L in [37] describes subset simulation approach using an adaptive Metropolis algorithm to extract rare failures events. The first level of the algorithm begins with the traditional MCS method and just few samples, after a performance function analysis, are kept and serve as “seeds” for Markov Chain in the adaptive Metropolis algorithm. The adaptive algorithm has some criteria that lets moving the system components to a rarer system state. Once the rare failures states are assessed, the probability indexes are determined by the product of the intermediate states’ conditioned probability moving to one state considering the conditioned and previous one.

In article [39] this adaptive algorithm proposed in [37] is extended to let application in components in the system following a discrete probability distribution. This paper applies subset simulation algorithm in Composite power system reliability.

PROPPE, C. in [41] presents the subset simulation and the “moving particle” as MCMC algorithms. Moving Particle is alike to subset simulation but in each step changes just the sample with the worst result in the performance function. PROPPE, C. compares both MCMC methods and shows that subset simulation requires more initial samples if compared to moving particles algorithm since most of particles are discarded during performance's analysis. However, the paper suggests that the moving particles algorithm must discard seeds used during the Markov Chain to avoid correlated samples while it is not necessary in subset simulation.

Contrary of PROPPE, C., HUA, B. *et al* in [39] indicates that seeds used in Markov Chain should be discarded.

In this thesis, a reliability adequacy will be studied using Monte Carlo Simulation and multi-area HL-I evaluation. The reliability evaluation will be conducted using Optimal Importance Sampling, Markov Chain Monte Carlo (MCMC), Stratification to represent VER and Monte Carlo to access the reliability indexes in a power electric system. This approach aims to improve efficiency and decrease computational simulation times while not losing accuracy in the reliability analysis.

In the literature there was not found studies applying all these techniques together to reduce the computational effort of the adequacy of a power system reliability.

3 Background Knowledge

3.1 System Modelling

Electrical power systems must guarantee the supply of energy, minimizing the occurrences of load shedding. So, reliability is important to evaluate the adequacy of electrical systems to supply the demand of energy. A reliability evaluation considering a single area system is computationally simple since the problem consists basically to guarantee that the total available power generation in a system is greater or equal than the demand of energy considering many different scenarios of load and generation.

However, a multi-area reliability evaluation requires considering the transmission lines connecting the different areas. A multi-area reliability analysis can be mathematically formulated by a linear programming trying to minimize the total load shedding considering the flow capacity among the areas. The mathematical formulation is indicated below:

$$Min \sum_{i=1}^{N} r_i$$

$$Sf + g + r = d$$

$$\underline{f} \leq f \leq \bar{f}$$

$$\underline{g} \leq g \leq \bar{g}$$

Where:

r is the loss of load, N is the number of areas, g is the total generation in each area, d the total demand of energy in the system, f is the power flow in the transmission lines between two areas, S is the reduced incidence matrix, $\underline{f}$ and $\bar{f}$ are the minimum and maximum flow capacity in each transmission line. $\underline{g}$ and $\bar{g}$ are the minimum and maximum generation in each area.

In addition, a multi-area system can be also expressed by graphs that consists in representing the system by nodes (or buses) connected by arcs. Graphs allow analyzing whether a scenario result or not in load shedding without solving an optimization problem.

3.1.1 Multi area systems represented by graphs

In the approach of using graphs to represent a multi area system, the nodes correspond to the areas in the system while the arcs correspond to the flow between these areas. The total available power of each area is represented by the arc capacity that leaves the node source (So) while the demand in each area is represented by the arc capacities that reaches the node terminal (T). The capacity between the areas are represented by flows in the arcs between the areas. Figure 9 represents a graph for a two-area system.

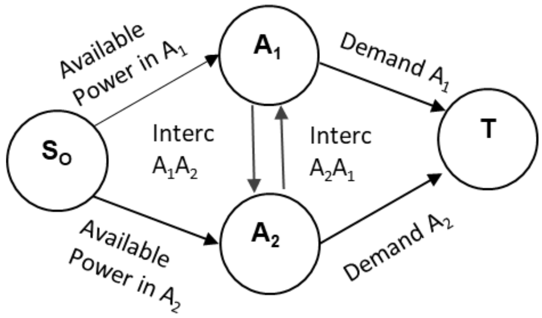


Figure 9: Graph for a two-area system

From the graph representation of an electrical system is possible to determine the maximum flow that can reach the demand and to conclude if it is possible to supply the demand of energy.

For instance, consider that the interconnection capacity between area 1 to area 2 and from area 2 to area 1 is 50 MW (each one). Suppose a scenario in which the total demand of area 1 is 80 MW and 100 MW in area 2. Consider that the total available power in area 1 is 150 MW while it is 40 MW in the area 2.

Since the total available capacity of generators in area 1 is 150 MW and the demand of area 1 is just 80 MW, 70 MW could be exported to area 2. However, the capacity of the transmission line between areas 1 and 2 is just 50 MW, and then, just 50 MW reaches area 2. Once the generation capacity in area 2 is 40 MW, is not possible to supply the total demand of area 2 (100MW), resulting in a 10 MW of load shedding.

The graph illustrating the maximum flow for this hypothetical scenario is illustrated in Figure 10.

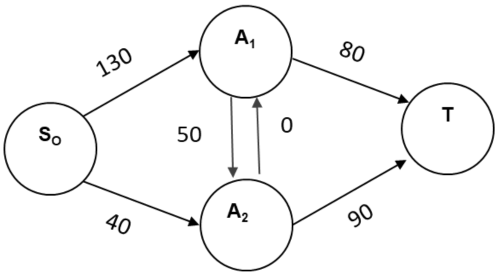


Figure 10: Maximum Flow in the graph for the two-area system example

It is a simple example and thereby could be solved graphically. The maximum flow can be determined analytically by solving a linear optimization problem which is indicated below:

$$Max \sum_{v=1}^{V} f_{v,T}$$

Subject to:

$$f_{v,w} \leq \overline{f_{v,w}} \quad \forall\,(v,w)\,\epsilon\,A$$

$$\sum_{w=1}^{V} f_{v,w} = \sum_{w=1}^{V} f_{w,v} \quad (\forall\,(v)\,\epsilon\,V - \{T, S_0\})$$

$$f_{v,w} \geq 0 \quad \forall\,(v,w)\,\epsilon\,A$$

For any node v or w in the graph, $f_{v,w}$ is the power flow between the arcs (n_v, n_w). A is the set of arcs in the system. To get the maximum flow from node source until node sink, the objective function consists in maximizing the flow from the nodes v that reaches the node sink $(f_{v,T})$. The first constraint indicates that the flow capacity in each arc of the graph $(f_{v,w})$ must be equal or lower than the limit of capacity of this arc $((f_{v,w}))$. The second constraint indicates that the total flow that reaches one node must be the same of the total flow leaving this same node (1st Kirchhoff Law), except for the nodes source and sink. Finally, the third constraint represents that the power flow in an arc cannot be negative.

For the same example used for illustrating the maximum flow in the graph, the mathematical formulation is indicated below:

$$Max\ f_{A_1,T} + f_{A_2,T}$$

Subject to:

$$f_{S_o,A_1} \leq 150;\ \ f_{S_o,A_2} \leq 40;\ \ f_{A_1,A_2} \leq 50;\ \ f_{A_2,A_1} \leq 50;\ \ f_{A_1,T} \leq 80;\ \ f_{A_2,T} \leq 90$$

$$f_{S_o,A_1} - f_{A_1,A_2} + f_{A_2,A_1} - f_{A_1,T} = 0$$

$$f_{S_o,A_2} + f_{A_1,A_2} - f_{A_2,A_1} - f_{A_2,T} = 0$$

$$f_{S_o,A_1} \geq 0; \quad f_{S_o,A_2} \geq 0; \quad f_{A_1,A_2} \geq 0; \quad f_{A_2,A_1} \geq 0; \quad f_{A_1,T} \geq 0; \quad f_{A_2,T} \geq 0$$

3.1.2 Maximum Flow – Minimum Cut Theorem

In 1962, it was demonstrated in [16] that determining the maximum flow in a graph is equivalent to obtain the cut in a graph with the minimum capacity between the arcs in the system. In other words, the maximum flow problem could be solved by its dual problem.

A cut in a graph corresponds to the minimum number of arcs that once removed from the graph separates the nodes source and sink, i.e, it's not possible to meet the demand of energy. The minimum cut is, consequently, the cut in which the sum of capacities of the arcs is minimum. The arcs in the minimum cut have the power flow equal to their maximum capacity.

Any cut in a graph can be the minimum cut depending on the scenario under analysis. In other words, the minimum cut depends on the dispatch scenario: generation dispatches, interconnection capacities and demand of energy. [17]

The main advantage for using the minimum cut approach to check if there is load supply is that it doesn't require solving an optimization problem for each scenario as is required in the maximum flow approach. It's possible to enumerate all possible cuts in a graph (2^N cuts where N is the number of areas in the graph) and for each scenario analyze which one is the minimum cut.

Figure 11 indicates for a two-area system the 4 possible feasibility cuts.

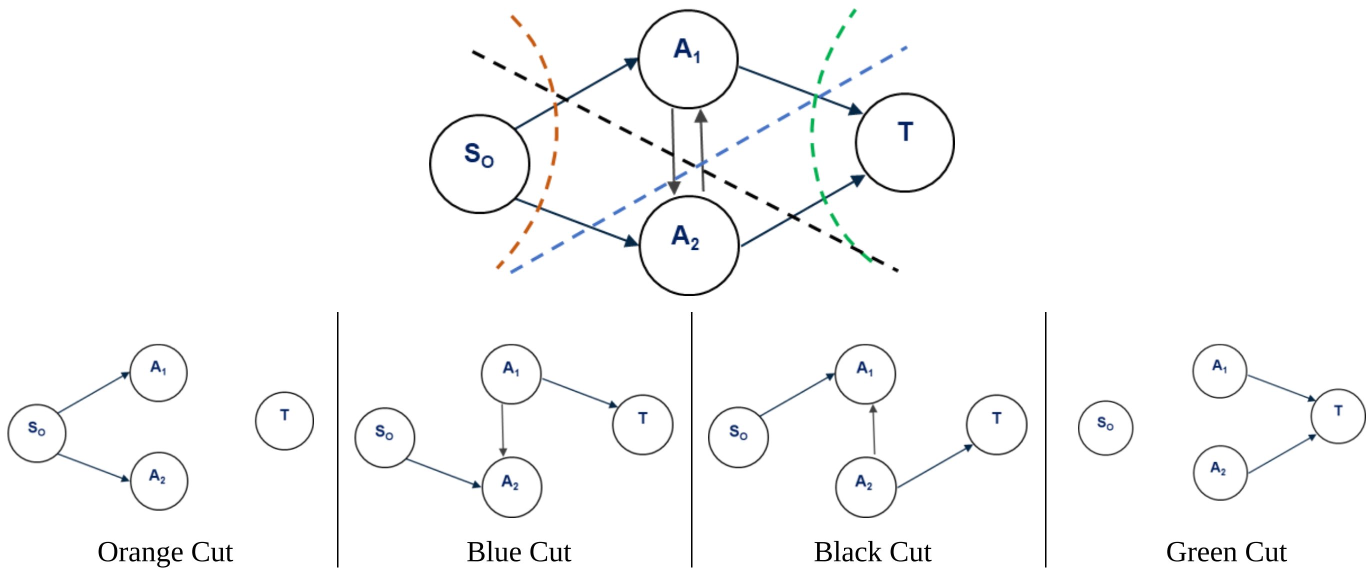


Figure 11: Feasibility cuts for a two-area system

If the green cut corresponds to the minimum cut in a graph it means that the demand arcs ($f_{A_1,T}$ and $f_{A_2,T}$) are at their maximum values and then, the demand of energy is being meet without load shedding. In this case the arc capacities reaching the sink are saturated, that is, the flow in the arcs is equal to the total demand to be supplied in the system.

Thereby, just three of the feasibility cuts result in load shedding since one of them (the green one) represents that it is not possible to flow energy from the source to the sink because all demand of energy has already been met. The inequalities corresponding to the infeasible cuts in a graph for a system with two areas are indicated below.

$$f_{S_o,A_1} + f_{S_o,A_2} \leq f_{A_1,T} + f_{A_2,T}$$

$$f_{S_o,A_2} + f_{A_1,A_2} + f_{A_1,T} \leq f_{A_1,T} + f_{A_2,T}$$

$$f_{S_o,A_1} + f_{A_2,A_1} + f_{A_2,T} \leq f_{A_2,T} + f_{A_1,T}$$

If at least one of these inequalities is met, there is load shedding in the system. The arcs in the left side of this inequality are the ones saturated. The first inequality corresponds to the cut indicated in orange in the Figure 11 and indicates that there is no available generation capacity to meet the demand of energy. The second inequality corresponds to the cut in blue and indicates that the demand in area 2 is not totally meet, while the third cut (in black) indicates that the demand of area 1 is not completely meet.

Furthermore, the cuts in a graph can be represented as indicated in Table 2, where "True" indicates that the arc of the graph is saturated.

Table 2: Enumeration of feasibility cuts in a two-area graph

Cut	f_{S_o,A_1}	f_{S_0,A_2}	f_{A_1,A_2}	f_{A_2,A_1}	$f_{A_1,T}$	$f_{A_2,T}$
1	True	True				
2		True	True		True	
3	True			True		True

For instance, consider the same values of the scenario used in the example for the maximum flow (demand of area 1 is 80 MW and 100 MW for area 2, and the interconnection capacity between areas equal to 50 MW and total available capacity of area 1 and 2 as 150 MW and 40 MW). It's possible to obtain the minimum cut referring to this graph by adding the capacities of the arcs present in each cut, as can be seen in the Table 3.

Table 3: Minimum cut for the two-area graph example

Cut	f_{S_o,A_1}	f_{S_0,A_2}	f_{A_1,A_2}	f_{A_2,A_1}	$f_{A_1,T}$	$f_{A_2,T}$	Total (MW)
1	150	40					190
2		40	50		80		170
3	150			50		100	300

For this scenario, the minimum cut corresponds to the cut 2, which has the sum of arc capacities 170 MW. From the maximum flow - minimum cut theorem, it's possible to conclude that the maximum flow is 170 MW. Since the total demand in the system is 180 MW, there is a deficit of 10 MW.

The minimum cut method requires enumerating a priori all possible cuts in a graph. Since the number of areas considered in reliability analyzes is generally small, the number of cuts to be enumerated is not large. In this thesis, the minimum cut approach is considered for calculating the energy supply because it has some analytical advantages that will be highlighted.

3.2 Reliability and Monte Carlo

The analysis of an electric system reliability comprehends to determine indices that indicates how reliable a system is. The commonly used indices are Loss of Load Probability (LOLP), Loss of Load Expectation (LOLE), Expected Power Not Supplied (EPNS), Expected Energy not Supplied (EENS), Loss of Load Frequency (LOLF) and Loss of Load Duration (LOLD).

These reliability indices are results of a reliability evaluation that consists of calculating the expected value of a function, which can be mathematically formulated as:

$$E(\phi(\underline{x})) = \sum_{\underline{x} \epsilon X} \phi(\underline{x}) P(\underline{x}) \tag{1}$$

Where X is a group of $\underline{x}$ vectors formed by the operative states of each element in the system (x_i) and $\phi(\underline{x})$ is the evaluation function which quantifies the effect of operative violations for each state $\underline{x}$. $P(\underline{x})$ is the probability of the sampled $\underline{x}$ state occurring.

A reliability analysis could be developed in an analytical way enumerating all possible operating states of a system and then, checking the states with load curtailment and their probabilities of occurrence. However, this approach might be used just for hypothetical systems since enumerating all states is quite laborious. For example, lets determine the loss of load probability of a system with 3 generators whose forced outage rate is 1%, 10 MW of installed capacity and load demand of 15 MW.

Table 4 indicates the operative state of generators and if it results in load shedding or not. “0” in the operative state indicates that the generators aren’t available, while “1” indicates the opposite. The symbol ✓ express load shedding and ✕ load supply.

Table 4: LOLP calculation by enumeration – three generators example.

Operative States			**Probability**	**Available Power**	**Load Curtailment**
G1 $(\boldsymbol{x_{G_1}})$	**G2** (x_{G_2})	**G3** (x_{G_3})			
0	0	0	0.01 x 0.01 x 0.01 = 1 E-6	0	✓
1	0	0	0.99 x 0.01 x 0.01 = 9.9 E-5	10	✓
0	1	0	0.01 x 0.99 x 0.01 = 9.9 E-5	10	✓
0	0	1	0.01 x 0.01 x 0.99 = 9.9 E-5	10	✓
1	1	0	0.99 x 0.99 x 0.01 = 9.8 E-3	20	✕
1	0	1	0.99 x 0.01 x 0.99 = 9.8 E-3	20	✕
0	1	1	0.01 x 0.99 x 0.99 = 9.8 E-3	20	✕
1	1	1	0.99 x 0.99 x 0.99 = 0.97	30	✕

Loss of Load Probability: 3 x 9.9 E-5 + 1 E-6 = 2.98 E-4

There are 2^{N_g} states in a system where N_g is the number of generators in the system. So, for a system with 3 generators, as the one in the example, there are 8 possible states. However, in a system with 10 generators, the number of states grows to 1024 states, while for a system with 100 generators there are more than 1E+30 possible states.

Then, the computational effort in enumerating the possible states of a system grows exponentially with the number of generators and it is possible to infer that this kind of analysis is impractical for real systems which are much larger and more complex having more elements and operative states.

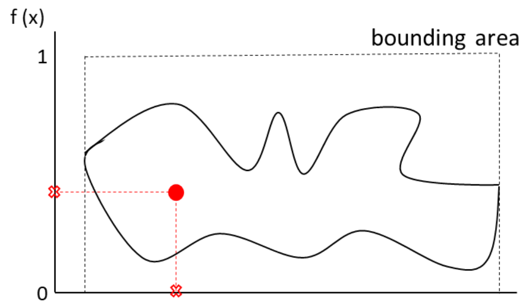
In case of generation reliability evaluation in which the reliability is evaluated without considering the transmission network, the convolution technique can be used since it allows analyzing a generation systems reliability with low computational effort. The convolution technique calculates the probability distribution of all system generators and once comparing with the load curve of the system lets analyze if there is a load curtailment in the system.

Although convolution is computationally efficient for generation reliability analysis, it is not an efficient technique to be applied when multi-area reliability analysis or generation and transmission analysis is performed, since the interconnection boundaries between areas or circuits limits should be considered. That is, when considering the transmission network, a reliability evaluation cannot be made comparing the total available generation and demand since the transmission network should be considered. So, it requires solving a linear optimization programming or a power flow.

Therefore, in a reliability evaluation of a generation and transmission network or even a multi-area reliability study, the most efficient technique to be applied is the Monte Carlo Simulation.

3.2.1 Monte Carlo Simulation - A brief of history

The credit for the invention of the Monte Carlo Simulation Method (MC) usually goes to Nicholas Metropolis and Stanislaw Ulam in the time of the Second World War, in a context that the nuclear bomb was been studied. MC basically proposes that one phenomenon can be studied considering some random samples that describe its behavior. Originally Monte Carlo Simulation was used to determine the integral of complicated regions by sampling pairs of random numbers. For example, consider that the integral of the below region should be determined.



The integral could be determined by Monte Carlo simulation sampling a random number in the axis x (according to the “domain of the function”) while, in the Y axis, other random number should be sampled following a uniform distribution resulting in a coordinate (red point inside the bounding area). Repeating this process an infinity of times, the integral of the function could be obtained by counting the number of coordinates that were inside the region and in bounding area. So, MC was commonly used to obtain a deterministic value which indicated the integral of a region. Later, this method began to be used for stochastic simulations and probability studies, such as, reliability studies.

3.2.2 Monte Carlo Simulation in Power Systems Reliability Evaluation

In contrast to the enumeration technique, Monte Carlo stochastic simulation is commonly used for the evaluation of a system reliability by sampling operating states of each element of the system. The samples are usually obtained considering each component probability distribution function indicating if the equipment is in the on mode, or under repair.

The algorithm of the Monte Carlo Simulation applied into reliability studies is characterized by sampling states of the components in the system and after sampling the state of each component, a vector containing the operating state of all components of the system is obtained. Then, for this vector is applied an evaluation function. Thus, the reliability indices are calculated based on the accumulated values of the evaluation function until a desired precision is reached.

So, consider m components in the system and a vector $\underline{x}$ comprehended by an operative state of each of these m components ($\underline{x} = x_1, x_2, \dots, x_m$). Consider ϕ as an evaluation function which is applied to the state $\underline{x}$ in order to verify if occurs load shedding in the system. In case of LOLP evaluation, $\phi(\underline{x})$ is an indicator function and then, $\phi(\underline{x}) = 1$ in case of load shedding and $\phi(\underline{x}) = 0$ otherwise. The expected value of this function $\phi(\underline{x})$ considering a set X of $\underline{x}$ vectors is:

$$E(\phi(\underline{x})) = \sum_{\underline{x} \epsilon X} \phi(\underline{x}) P(\underline{x}) = \Phi$$

However, is not possible to evaluate all possible states ($\underline{x}$) in real system because it would imply considering enumerating all states. Nevertheless, $P(\underline{x})$ could be replaced by the frequency at which the state $\underline{x}$ is sampled in X.

$$\Phi = \sum_{\underline{x} \epsilon X} \phi(\underline{x}) \frac{n(\underline{x})}{\text{N}}$$

So, the expected value of $E(\phi(\underline{x}))$ can be determined considering N samples as:

$$\overline{E}(\phi(\underline{x})) = \frac{1}{\mathrm{N}} \sum_{j}^{\mathrm{N}} \phi\left(\underline{x_j}\right) = \overline{\Phi}$$

Where $\phi\left(\underline{x_j}\right)$ is the evaluation function applied to $\underline{x_j}$. It's important to highlight that $\overline{E}(\phi(\underline{x}))$ is an estimate of the real value of $E(\phi(\underline{x}))$. The variance of the expected value of the evaluation function $\phi(\underline{x})$ can be measured as:

$$Var\,(\overline{\Phi}\,) = \frac{1}{\mathrm{N}}\, Var(\Phi)$$

This equation can be rewritten considering the standard deviation (σ):

$$\sigma(\overline{\Phi}\,) = \frac{\sigma(\Phi)}{\sqrt{\mathrm{N}}}$$

Dividing both sides by Φ $\left(\Phi = E\left(\phi(\underline{x})\right)\right)$ and considering α as the coefficient of variation (relation between the standard deviation and the average value of the evaluation function), the following is obtained:

$$\alpha = \frac{\sigma(\overline{\Phi})}{E\left(\phi(\underline{x})\right)} = \frac{\sigma(\Phi)}{\sqrt{\mathrm{N}}\; E\left(\phi(\underline{x})\right)}$$

Then, considering that the accuracy of the Monte Carlo Simulation can be measured by the coefficient of variation, the number of samples required by MC to achieve this α accuracy is:

$$\alpha^2 = \frac{\left(\sigma\,(\overline{\Phi})\right)^2 \mathrm{N}^2}{E\left(\phi(\underline{x})\right)^2} = \frac{Var(\Phi)\,\mathrm{N}^2}{E\left(\phi(\underline{x})\right)^2} = \frac{Var(\overline{\Phi})\mathrm{N}}{E\left(\phi(\underline{x})\right)^2}$$

$$\boxed{\mathrm{N} = \frac{Var(\overline{\Phi})}{E\left(\phi(\underline{x})\right)^2 \alpha^2}} \qquad (2)$$

From the formula above is possible to conclude that the number of samples required to get a precision of α for the reliability indexes is direct proportional to the variance of $\overline{E}(\phi(\underline{x}))$, an estimate of the real value of $E(\phi(\underline{x}))$, and invert proportional to the squared of the real value expected value. Moreover, this formula lets concluding that MC can be applied to any system regardless of its size, complexity or number of states, since for a given accuracy (α) the number of samples required just varies with the variance of the expected value of the evaluation function.

In case that the evaluation function is the Loss of Load Probability (LOLP), $\phi(\underline{x})$ is an indicator function since it can assume a zero value if there's no load shedding or 1 if the opposite happens. Moreover, since the evaluation function is an indicator function it follows a binominal distribution and the variance of $\phi(\underline{x})$ can be written as $Var\left(\phi(\underline{x})\right) = \overline{E}\left(\phi(\underline{x})\right)\left(1 - \overline{E}\left(\phi(\underline{x})\right)\right)$.

For real systems the expected value of load shedding is small since electric power systems are in general reliable. So, is usual to consider that $Var\left(\phi(\underline{x})\right) \sim \overline{E}\left(\phi(\underline{x})\right)$ and then the number of samples required for an α accuracy can be simplified to:

$$N = \frac{1}{\alpha^2 \overline{E}\left(\phi(\underline{x})\right)} = \frac{1}{\alpha^2\, LOLP} \tag{3}$$

So, we are led to conclude that MC is a powerful method since it lets considering stochasticity in the sampling simulation process and can be applied regardless of the number of states in a system which lets being used for reliability studies including the transmission network. So, these characteristic makes MC more efficient if compared to the enumeration method and to the convolution technique.

Nevertheless, since electric systems are usually reliable, the number of samples required for a given accuracy is usually very big which results in a great computational effort.

4 Relevant Techniques for the Proposal

4.1 Importance Sampling

Monte Carlo Simulation Method corresponds to a stochastic method for the power reliability assessment since lets considering different operating scenarios for calculating reliability indexes. The vectors sampled through the Standard Monte Carlo Simulation are composed by the operating states of components in the electric system (generators and transmission lines). Nevertheless, electrical power systems are usually reliable, and then the probability of sampling states that result in load shedding is very small. Therefore, for calculating accurate reliability indexes, with a small coefficient of variation (α), lot of samples should be drawn through the Monte Carlo Simulation.

From Equation (2) it can be concluded that the number of samples required to get a precision of α in the reliability indexes is direct proportional to the variance of the expected value of the evaluation function. Thereby, for the sake of reducing the number of samples required for a reliability analysis and for reducing the computational effort, variance reduction techniques can be considered to reduce the variance of the evaluation function. However, applying these techniques can not compromise the accuracy of the index estimation neither the values of the reliability indexes of a system. Since an estimator's variance is directly proportional to the number of samples, reducing the estimator's variance will reduce the number of samples and time spent during computational simulations.

Importance sampling (IS) is a variance reduction technique that aims to skew the probability of components failure in order to increase the probability of the ones that more contribute to load shedding scenarios and to reduce the probabilities of the components that result in load supply scenarios. In this way, IS lets obtaining "tilted" probabilities that turn easier to obtain scenarios that result in load shedding, speeding up the reliability analysis.

As mentioned in the previous chapter, the Monte Carlo Simulation was not developed with the propose of reliability assessment. Moreover, it didn't require considering any probability distribution. Just as Monte Carlo Simulation began to be applied in reliability studies the probability of the elements was usually considered.

Nevertheless, instead of drawing samples from the original probability distribution $P(x)$, they could be drawn from another distribution $P^*(x)$ that increases the probability of sampling scenarios with load shedding, *but not changing the expected value* *of the reliability estimator*. If the Equation (2) in multiplied and divided by $P^*(x)$, the expected value of LOLP will not change, as indicated in Equation (4).

$$E(\phi(\underline{x})) = \sum_{x \epsilon X} \left[\frac{\phi(\underline{x}) P(\underline{x})}{P^*(\underline{x})}\right] P^*(\underline{x}) \tag{4}$$

Comparing Equations (2) and (4), $\phi^*(\underline{x})$ can be defined as:

$$\phi^*(\underline{x}) = \frac{\phi(\underline{x})\, P(\underline{x})}{P^*(\underline{x})} \tag{5}$$

This is the idea behind the importance sampling technique: increase the probability of sampling vectors ($\underline{x}$) that results in load curtailment, i.e., $\phi(\underline{x}) = 1$.

It´s intuitive that one possible approach for increasing the probability of sampling vectors resulting in load shedding should be increasing the failure rate of components in the power systems, such as transformers and transmission lines.

On one hand, it would be difficult to identify the equipment that, in failing, contributes to the failure states of the system. For instance, if a generator has a failure rate of 1%, and it is broken ($\phi(x) = 1$) in 10% of scenarios that result in load shedding it means that it’s very disruptive to system operation. However, if that same generator is out of operation in 1% of the scenarios that result in non-supply of the energy demand, it not much contributes to the load shedding and therefore its probability of failure should not be “tilted” by the IS.

One the other hand, assuming at first that the generators whose failures more contribute to load shedding scenarios in the system are known a priori, it would be hard to identify how to skew these generators’ failure rate to properly reduce the LOLP variance.

Hence, it’s difficult to determine the way to distort the failure rate of the system components to obtain the new probability density function ($P^*(\underline{x})$) that reduces the variance of the LOLP estimator.

Suppose $P^*(\underline{x})$ is defined as $P^*(\underline{x}) = \frac{\phi(\underline{x})P(\underline{x})}{E(\phi(\underline{x}))}$. Substituting $P^*(\underline{x})$ in Equation (4) it’s possible to conclude that the expected value of the LOLP estimator can be determined with just one sample and it’s calculated regardless of the sampled vector $\underline{x}$.

$$E(\phi(\underline{x})) = \sum_{\underline{x} \epsilon X} \left[\frac{\phi(\underline{x}) P(\underline{x})}{\left(\frac{\phi(\underline{x}) P(\underline{x})}{E(\phi(\underline{x}))}\right)}\right] P^*(\underline{x})$$

$$E(\phi(\underline{x})) = \sum_{\underline{x} \epsilon X} E(\phi(\underline{x}))\, P^{*}(\underline{x}) \tag{6}$$

In case of this $P^{*}(\underline{x})$, just one sample is required to determine the LOLP estimator which means that the variance of the LOLP estimator is null.

However, it's obvious that $P^{*}(\underline{x})$ defined as $\frac{\phi(\underline{x})P(\underline{x})}{E(\phi(\underline{x}))}$ is a theoretical probability density function since it is function of $E(\phi(\underline{x}))$ which is the value to be determined. Despite being a theoretical distribution, it indicates how powerful the tool of importance sampling is to analyze the reliability of a power system since lets reducing the variance of reliability indexes.

4.1.1 Example of Importance Sampling with Enumeration

This section aims to present an example of reliability analysis applying the concepts of importance sampling that will be used throughout this work.

For the sake of understanding the concept and potential of importance sampling, a small and simple example is presented. Consider a theorical system with 5 generators, each one with an available power capacity of 3 MW and the demand of energy in the system equal to 8 MW. So, at least 3 generators should fail to result in load shedding.

Table 5: Hypothetical System Configuration

G_1	G_2	G_3	G_4	G_5	Load
3 MW	3 MW	3 MW	3 MW	3 MW	8 MW

At first, supposes that all 5 generators have the failure rate of 3%. The LOLP can be determined enumerating all possible states.

Table 6: Squared deviation from the mean and expected value for generators with same forced failure rate - Enumeration method -

Number of unavailable generators	Non-viability indicator function $\boldsymbol{\phi}(\underline{\boldsymbol{x}})$	Outage probability (State enumeration) $P(\underline{x})$	$\boldsymbol{\phi}(\underline{\boldsymbol{x}})\ \boldsymbol{P}(\underline{\boldsymbol{x}})$	$\boldsymbol{\phi}^2(\underline{\boldsymbol{x}})\ \boldsymbol{P}(\underline{\boldsymbol{x}})$
0	0	$(0.97)^5 = 0.8587$	0	0
1	0	$\binom{5}{1}\ (0.97)^4(0.03)^1 = 0.1328$	0	0
2	0	$\binom{5}{2}\ (0.97)^3(0.03)^2 = 0.0082$	0	0
3	1	$\binom{5}{3}\ (0.97)^2(0.03)^3 = 0.00025$	2.54E-04	2.54E-04
4	1	$\binom{5}{4}\ (0.97)^1(0.03)^4 =$ 3.93E-06	3.93E-06	3.93E-06
5	1	$(0.03)^5 =$ 2.43E-08	2.43E-08	2.43E-08
			$E\left(\phi(\underline{x})\right) =$ 2.58E-04	$E\left(\phi^2(\underline{x})\right)$ = 2.58E-04

Remembering that:

$$Var\ \left(\phi(\underline{x})\right) = E\left(\phi^2(\underline{x})\right) - \left(E\left(\phi(\underline{x})\right)\right)^2 \qquad (7)$$

The variance of $\phi(\underline{x})$ is 2.58 E-04 [6].

From Table 6 is possible to observe that the estimator's variance is big since it has the same magnitude order if compared to the expected value.

Now consider that the probability of failure of the generators is distorted and increased to 10%. Table 7 describes the calculation of loss of load probability and its variance applying the concepts and formulas of importance sampling.

[6] Since $E\left(\phi(\underline{x})\right)$ is a small value, the $Var\ \left(\phi(\underline{x})\right) = E\left(\phi^2(\underline{x})\right) - \left(E\left(\phi(\underline{x})\right)\right)^2$ is approximately equal to $Var\ \left(\phi(\underline{x})\right) = E\left(\phi^2(\underline{x})\right)$

Table 7: Increasing failure rate of generators to 10%

Number of unavailable generators	Non-viability indicator function $\phi(\underline{x})$	Distorted Outage probability (State enumeration) $P^*(x)$	$\phi^*(\underline{x})$	$\phi^*(\underline{x})P^*(\underline{x})$	$\phi^{*2}(\underline{x})P^*(\underline{x})$
0	0	0.5905	0	0	0
1	0	0.3280	0	0	0
2	0	0.0729	0	0	0
3	1	0.0081	0.0314	2.54 E-04	7.97 E-06
4	1	4.5 E-04	8.73E-03	3.93 E-06	3.43 E-08
5	1	1 E-05	2.43E-03	2.43 E-08	5.90 E-11
				$E\left(\phi(\underline{x})\right)$ = 2.58E-04	$E\left(\phi^2(\underline{x})\right)$ = 8.00E-06

Considering the Equation (7), the variance of $\phi(\underline{x})$ is calculated. Var $\left(\phi(\underline{x})\right)$ is equal to 7.94E-06.

Comparing Table 5 and Table 6 is possible to observe that the expected value of LOLP $\left(E\left(\phi(\underline{x})\right)\right)$ didn't change, but the variance reduced when the failure rate of generators increased.

Nevertheless, if the failure rate of the generators is progressively increased, it is observed that at a given moment the variance of the estimator increases rather than decreases, indicating that there is a *minimum value* to which the variance can be reduced. This value from which the variance starts to increase correspond to the optimum value, i.e., it is the distorted failure rate of the generators that would minimize the variance of the LOLP estimator. The graph in Figure 12 illustrates, for this hypothetical system, the variance values for different generators failure rates.

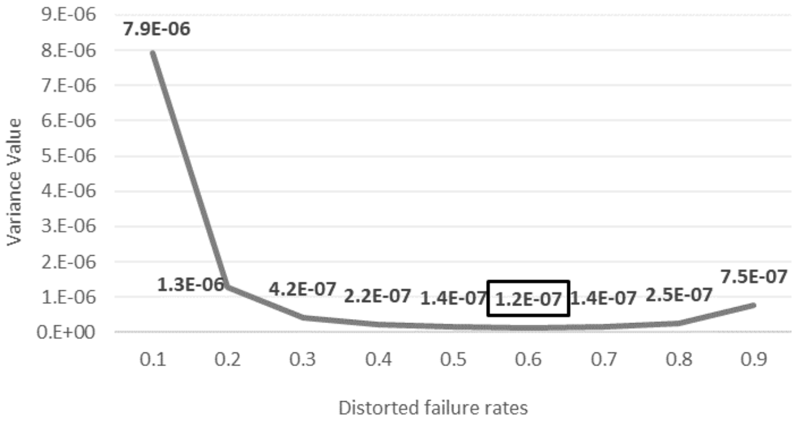


Figure 12: Variance values for different forced failure rates

It's notable that there is a distorted (or "tilted" failure rate) in which the LOLP variance value is minimum. From this value, if the generator failure rate is increased, the LOLP variance value increases instead of decreasing.

The graph in Figure 13 illustrates the comparison between the number of samples required for determining the LOLP expected value for many generators' failure rate, considering a coefficient of variation (α) of 5%. Considering the original failure rate (3%), the reliability assessment requires more than 1.5 million samples.

Nevertheless, skewing the original failure rates, increasing the probability of failure, the number of Monte Carlo samples reduces, as illustrated in Figure 13.

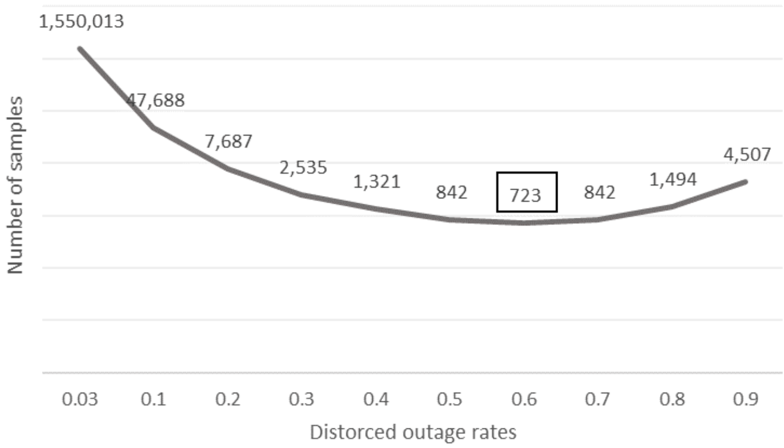


Figure 13: Number of samples required by MC

Figure 14 illustrates the speed up in Monte Carlo Simulation whether using the Monte Carlo Standard approach or Importance Sampling technique.

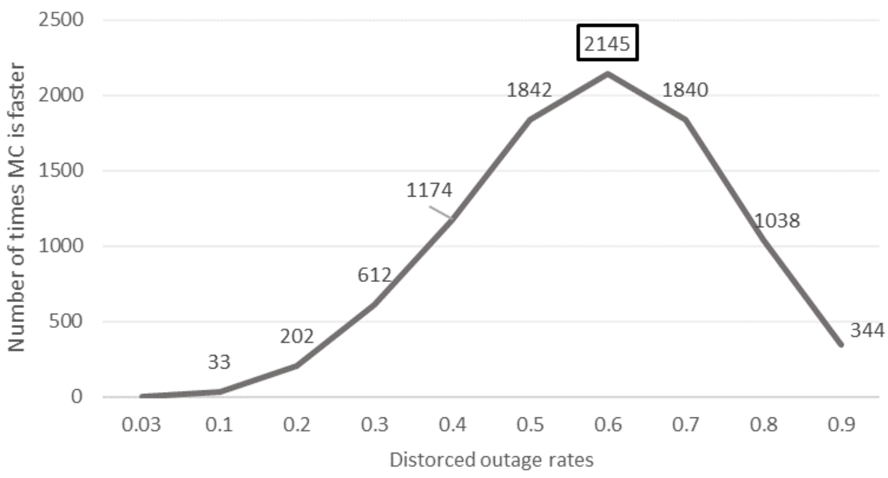


Figure 14: Speed up in Monte Carlo Simulation

In this example the importance sampling optimization was applied considering that all components in the system had the same failure rates. This example points out that there is an *optimum value* in which the estimator variance is *minimum*. Since it's a small and hypothetical system, with all components with the same failure rate, it's possible to enumerate all states and test many different failure rates to get the optimum one.

However, in case that generators may have different failure rates it would not be efficient to try to find the optimal distortion for each generator separately, because a scenario with load shedding depends on the failure rate of all components in the system. In other words, the electrical power reliability assesses the load supply in the electrical system considering the operation state of all components together.

In addition, electric power systems are usually complex and with lots of components, such as generators, transformers and transmission lines and each of them may have different failure rates. Therefore, in this case, the optimal IS can't be determined by an analytical approach.

4.1.2 Improving Monte Carlo Simulation efficiency

Although the Monte Carlo Simulation method applied to power systems commonly uses the probability distribution function of each component of the system to sample the operating state of each component individually, the existence or not of load shedding depends on the operating state of all system components.

Thus, for a multi area system with many interconnected electrical areas, one way to improve the efficiency of the Monte Carlo simulation would be to consider the distribution of the total available power of each area (obtained by convolution techniques) and sample directly from the distribution function of the available power of each area. Therefore, the efficiency of MCS is improved considering sampling from the *total available power distribution of each area* and as will be presented in further chapters it makes easier to apply the methods that will be proposed.

So, for sampling a total available power in each area, the Inverse Transform Method can be applied. The Inverse Transform Method is illustrated in Figure 15 and states that random samples of a variable X can be obtained applying uniformly distributed variables and in the inverse of the cumulative probability distribution $X = F_X^{-1}\,(U)$, where U~[0,1].

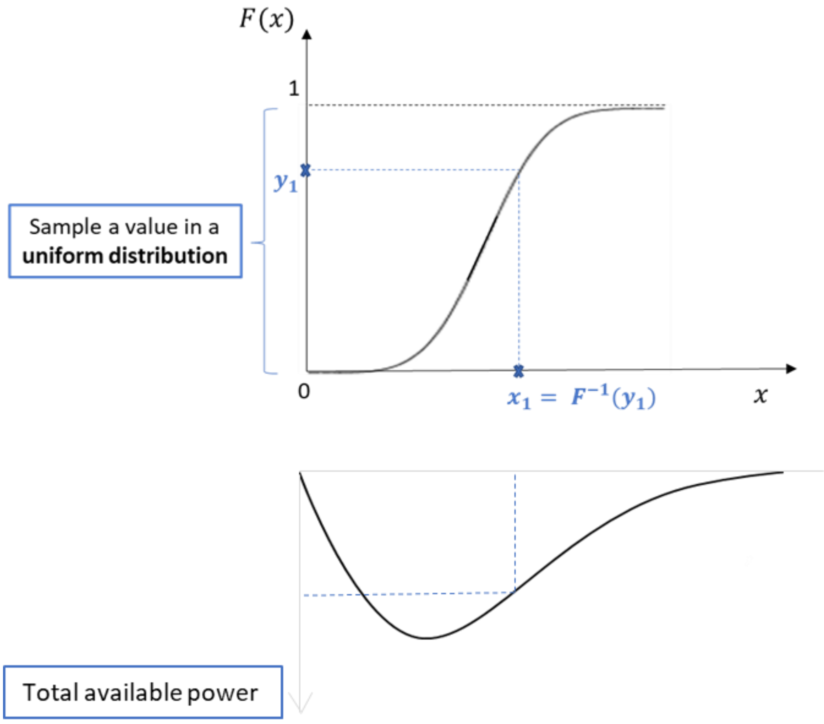


Figure 15: Inverse Transform Method

In Figure 15, x_1 corresponds to a sample containing the area available power.

4.1.3 Importance Sampling Optimization

Importance sampling is a technique of variance reduction that proposes obtaining a "new distribution function" in which is more likely to sample load shedding scenarios, reducing the variance of the estimator of the evaluated index in the reliability analysis. However, there is an optimum value in which the estimator variance is minimum, reducing the number of samples required for Monte Carlo Simulation (for a given accuracy value).

For the sake of determining this optimum value it is proposed an IS optimization method which can be applied from a sample of *independent draws* resulting in load shedding.

However, to obtain samples with failure states that result in no-load supply is the difficulty of analyzing the reliability of electrical systems. Sampling these draws require great computational effort running a Monte Carlo Simulation which is costly since electric systems are usually reliable with small loss of load probability.

Thereby, considers, at first, that these samples are known a priori. Later, in this thesis (chapter 4) will be proposed a methodology to improve the approach of generating samples resulting in system's load shedding.

Let $\{\underline{x}^s, s = 1, \dots, S\}$ be a set of S vectors containing generators' states that result in load curtailment. Each vector $\underline{x}^s$ has I components $\{x_i^s, i = 1, \dots, I\}$, where each component corresponds to the state of the generator in the draw s: $x_i^s = 1$ (broken) or $x_i^s = 0$ (running, i.e., operating).

The goal of IS optimization is to maximize the probability of sampling vectors of generation states that lead to load shedding. This is equivalent to determine the failure probability parameter of each generator $\{p_i^*, i = 1, \dots, I\}$ which maximizes the probability of sampling the vector $\{\underline{x}^s, s = 1, \dots, S\}$. This corresponds to the maximum likelihood estimation of the sample. Since the draws are independent, the joint probability of the draws is given by the product of the probabilities of each draw.

$$\max_{\{p_i^*\}} \prod_s P^*(\underline{x}^s) \tag{8}$$

Where $P^*(\underline{x^s})$ is the vector with the distorted failure probability of each generator.

This maximization problem is usually solved applying the logarithmic function that is an increasing function. Then, the optimization problem can be rewritten as:

$$\max_{\{p_i^*\}} \sum_s \log\left(P^*(\underline{x}^s)\right) \tag{9}$$

On the other hand, $P^*(\underline{x}^s)$ can be written as the product of each component probability failure. Then:

$$P^*\left(\underline{x}^s\right) = \prod_i p^*(x_i^s) \tag{10}$$

$p^*(x_i^s) = p_i^*$ if $x_i^s = 1$ and then, the equipment is broken in the sample s and $p^*(x_i^s) = 1 - p_i^*$ if $x_i^s = 0$.

Then, applying the logarithmic function at Equation (10) and substituting in Equation (9), the optimization problem is:

$$\max_{\{p_i^*\}} \sum\nolimits_s \sum\nolimits_i \log\big(p^*(x_i^s)\big) \tag{11}$$

Changing the order of the sums and considering the definitions above:

$$\max_{\{p_i^*\}} \sum\nolimits_i \sum\nolimits_s \log\big(p^*(x_i^s)\big) \tag{12}$$

$$\sum\nolimits_i \max_{\{p_i^*\}} \left[\sum_{x_i^s=1} \log(p_i^*) + \sum_{x_i^s=0} \log(1-p_i^*)\right] \tag{13}$$

$$= \sum\nolimits_i \max_{\{p_i^*\}} [S_i^1 \log(p_i^*) + (S - S_i^1) \log(1-p_i^*)] \tag{14}$$

Where S is the total number of samples, S_i^1 is the number of samples in which the component *i* is broken, i.e., $x_i^s = 1$.

Once deriving and equating Equation (14) to zero one obtains:

$$\frac{S_i^1}{p_i^*} - \frac{(S - S_i^1)}{(1-p_i^*)} = 0$$

$$S_i^1(1-p_i^*) = (S - S_i^1)p_i^*$$

$$p_i^* = \frac{S_i^1}{S} \tag{15}$$

From Equation (15) is possible to conclude that the optimum value for the “tilted” failure rate of each component corresponds to the frequency in which the generators were broken in the sample S. Thereby, Equation (15) corresponds to the optimum IS, or in other words, it indicates the “tilted” probabilities that let minimizing the variance of the reliability index estimator.

The following sections will present examples with the application of the IS optimization using samples of independent draws. These examples consider the same hypothetical system of 5 generators each one having a maximum available power of 3MW and the demand to be met is 8 MW.

4.1.3.1 Examples of IS Optimization applied to each generator individually

This section applies the IS optimization to “tilt” the original probability distribution of each generator individually. In the first example all generators have the same failure rate, while in the second example, generators have different rates of failure.

Generators with same failure rates

Consider the 5 generators each one with a failure rate of 3% (Table 5). Applying the Monte Carlo Standard Simulation, 10^6 samples are drawn, and 261 samples result in load shedding. From these 261 samples, it's observed the number of times each component has failed, and then from the Equation (15) the distorted failure rate of these components is determined.

Table 8: "Tilted" failure rate - IS optimization - generators with same failure rate

Generators	Number of failures	$P(x_i)$	$P^*(x_i)$
G_1	173	0.03	0.66
G_2	142	0.03	0.54
G_3	161	0.03	0.62
G_4	155	0.03	0.59
G_5	154	0.03	0.59

It's worth pointing out that the distorted failure rate of generators obtained from the IS optimization is very similar to the one that results in the minimum variance estimator calculated considering enumeration method, shown in section 4.1.1. (Optimum "tilted" probability = 0.6 and a minimum variance of 1.2E-7 as shown in Figure 12 and Figure 13).

In this example of generators with the same failure rates, applying the IS optimization results in a variance of 1.20E-07, as indicated in Table 9.

Table 9: IS using samples - Generators with same failure rate

Number of unavailable generators	Non-viability indicator function $\phi(\underline{x})$	Distorted Outage probability (State enumeration) P*($\underline{x}$)	$\phi^*(\underline{x})$	$\phi^*(\underline{x})P^*(\underline{x})$	$\phi^{*2}(\underline{x})P^*(\underline{x})$
0	0	0.0098	0	0.00E+00	0.00E+00
1	0	0.0752	0	0.00E+00	0.00E+00
2	0	0.2292	0	0.00E+00	0.00E+00
3	1	0.3469	7.32E-04	2.54E-04	1.86E-07
4	1	0.2609	1.51E-05	3.93E-06	5.92E-11
5	1	0.0779	3.12E-07	2.43E-08	7.58E-15
				$E\left(\phi(\underline{x})\right) =$ 2.58 E-04	$E\left(\phi^2(\underline{x})\right) =$ 1.20 E-07

Considering the Equation (6), the variance of $\phi(\underline{x})$ is calculated. Var $\left(\phi(\underline{x})\right)$ is equal to 1.20E-07. Thereby, since the variance reduced from 2.58E-04 to 1.20E-07, applying IS results in a speed up of more than 2150 times.

Generators with different failure rates

IS optimization can be even used if the components have different failure rate. So, suppose for this hypothetical example, that generators instead of having a failure rate of 3% have failure rates of 1%, 2%, 3%, 4% and 5% respectively.

From the Monte Carlo Standard simulation 10^6 samples are drawn in which 237 results in load shedding. From these 237 samples, it's observed the number of times each component was failed, and then from the Equation (15) the distorted failure rate of these components is determined.

Table 10: "Tilted" failure rate - IS optimization - generators with different failure rate

Generators	Number of failures	$P(x_i)$	$P^*(x_i)$
G_1	70	0.01	0.30
G_2	137	0.02	0.58
G_3	152	0.03	0.64
G_4	172	0.04	0.73
G_5	184	0.05	0.78

At first, note that importance sampling aims to change the failure probability of the system components, "increasing" the failure rates of the components that more contribute to the load curtailment in the system, i.e., those that are broken more frequently in load shedding scenarios. Those that contribute most to non-demand supply are those that have the most increased failure probability.

For the sake of analyzing the variance and expected value of the LOLP estimator value considering the original failure rates (P) and the distorted ones (P^*) , it's used the enumeration method for calculating these values for generators with different failure rates. The expected value using the Standard Monte Carlo Simulation, or the Importance Sampling technique is the same and it's equal to 2.17E-04 (Table 11). Nevertheless, the LOLP estimator's variance reduces from 2.17E-04, in the original case, to 7.61E-08 applying Importance Sampling, as indicated in Table 12. Since the variance reduced from 2.17E-04 to 7.61E-08, applying IS results in a speed up of more than 2850 times.

It corroborates the efficacy of the IS methodology even if applied in electrical systems with components with different failure rates.

Table 11: Squared deviation from the mean and expected value for generators with different failure rate - Enumeration method -

Number of unavailable generators	Non-viability indicator function $\boldsymbol{\phi}(\underline{\boldsymbol{x}})$	Outage probability (State enumeration) $P(\underline{x})$	$\boldsymbol{\phi}(\underline{\boldsymbol{x}})\,\boldsymbol{P}(\underline{\boldsymbol{x}})$	$\boldsymbol{\phi}^{2}(\underline{\boldsymbol{x}})\,\boldsymbol{P}(\underline{\boldsymbol{x}})$
0	0	0.86	0	0
1	0	0.13	0	0
2	0	0.01	0	0
3	1	2.14 E-04	2.14 E-04	2.14 E-04
4	1	2.68 E-06	2.68 E-06	2.68 E-06
5	1	1.20 E-08	1.20 E-08	1.20 E-08
			$E\left(\phi(\underline{x})\right) =$ 2.17E-04	$E\left(\phi^{2}(\underline{x})\right) =$ 2.17E-04

Considering the Equation (6), the variance of $\phi(\underline{x})$ is calculated. Var $\left(\phi(\underline{x})\right)$ is equal to 2.17E-04.

Table 12: IS using samples - Generators with different failure rate

Number of unavailable generators	Non-viability indicator function $\boldsymbol{\phi}(\underline{\boldsymbol{x}})$	Distorted Outage probability (State enumeration) $P^*(\underline{x})$	$\boldsymbol{\phi}^{*}(\underline{\boldsymbol{x}})$	$\boldsymbol{\phi}^{*}(\underline{\boldsymbol{x}})\boldsymbol{P}^{*}(\underline{\boldsymbol{x}})$	$\boldsymbol{\phi}^{*2}(\underline{\boldsymbol{x}})\boldsymbol{P}^{*}(\underline{\boldsymbol{x}})$
0	0	0.0065	0	0	0
1	0	0.0634	0	0	0
2	0	0.2279	0	0	0
3	1	0.3726	5.75 E-04	2.14 E-04	1.23 E-07
4	1	0.2678	1.00 E-05	2.68 E-06	2.68 E-11
5	1	0.0617	1.94 E-07	1.20 E-08	2.33 E-15
				$E\left(\phi(\underline{x})\right) =$ 2.17E-04	$E\left(\phi^{2}(\underline{x})\right) =$ 1.23E-07

Considering the Equation (6), the variance of $\phi(\underline{x})$ is calculated. Var $\left(\phi(\underline{x})\right)$ is equal to 7.61E-08.

4.1.3.2 Examples of IS Optimization applied to the total available power

This section aims to apply the IS optimization to the total available power capacity in the system. So, instead of "tilting" the failure rate of each component, it "tilts" the marginal distribution of the *available power of each area*.

In the first example all generators have the same failure rate, while in the second example, generators have different failure rates.

By Monte Carlo Simulation, 10^6 vectors are drawn with the available capacity of each generator. Summing the available power of each vector, the total available power is obtained. Considering the total available power is possible to determine the number of available generators in the system since each generator follows a Bernoulli distribution.

<u>Generators with same failure rate (3%)</u>

For this hypothetical system with all generators with a failure rate of 3%, the expected value and variance of LOLP estimator is 2.58E-04 as indicated in Table 6.

Table 13: Association of total available power and load shedding (generators with same failure rates)

Available Power (MW)	**Number of unavailable generators**	**Frequency of total available power**
0	5	0
3	4	2
6	3	259
9	2	8,271
12	1	132,299
15	0	859,169

Since load shedding occur if three or more generators fail, importance sampling technique aims to increase the outage probability of three, four or five generators failing. The distorted probabilities of the total available power can be determined considering the proportion of draws with three or more generators under repair in the total number of draws that result in load curtailment.

Table 14: IS optimization - Total available power - Generators same failure rate

Number of unavailable generators	Distorted Outage probability $P^*(\underline{x})$	$\boldsymbol{\phi}^*(\underline{\boldsymbol{x}})$	$\boldsymbol{\phi}^*(\underline{\boldsymbol{x}})\boldsymbol{P}^*(\underline{\boldsymbol{x}})$	$\boldsymbol{\phi}^{*2}(\underline{\boldsymbol{x}})\boldsymbol{P}^*(\underline{\boldsymbol{x}})$
5	0	0	0	0
4	0.0077	0.005	3.93E-06	2.01E-09
3	0.9923	0.003	2.54E-04	6.50E-08
			$E\left(\phi(\underline{x})\right) =$ 2.58 E-04	$E\left(\phi^2(\underline{x})\right) =$ 6.71 E-08

Considering the Equation (6), the Var $\left(\phi(\underline{x})\right)$ is equal to 5.01E-10, $Var\left(\phi(\underline{x})\right) \sim 0$.

<u>Generators with different failure rate</u>

Now suppose for this hypothetical example that generators instead of having a failure rate of 3% have failure rates of 1%, 2%, 3%, 4% and 5% respectively. The LOLP expected value and the variance of LOLP estimator is 2.17E-04 and 7.61E-08 as indicated in Table 12.

Table 15: Association of total available power and load shedding (generators with different failure rates)

Available Power (MW)	Number of unavailable generators	Frequency of total available power
0	5	0
3	4	4
6	3	233
9	2	7,850
12	1	133,728
15	0	858,158

The distorted probabilities of the total available power can be determined using the same approach used in the case that generators had the same failure rate.

Table 16: IS optimization - Total available power - Generators different failure rate

Number of unavailable generators	Distorted Outage probability P*($\underline{x}$)	$\boldsymbol{\phi}^*(\underline{\boldsymbol{x}})$	$\boldsymbol{\phi}^*(\underline{\boldsymbol{x}})\boldsymbol{P}^*(\underline{\boldsymbol{x}})$	$\boldsymbol{\phi}^{*2}(\underline{\boldsymbol{x}})\boldsymbol{P}^*(\underline{\boldsymbol{x}})$
5	0	0	0	0
4	0.0168	1.59E-04	2.68E-06	4.26E-10
3	0.9831	2.18E-04	2.14E-04	4.67E-08
			$E\left(\phi(\underline{x})\right)=$ 2.17 E-04	$E\left(\phi^2(\underline{x})\right)=$ 4.71 E-08

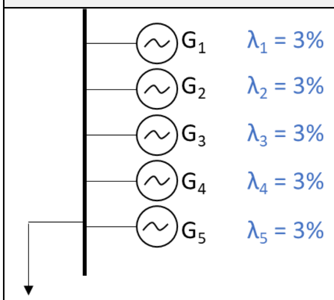
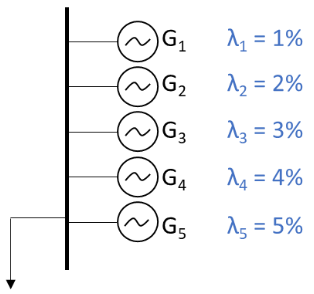
Considering the Equation (6), the Var $\left(\phi(\underline{x})\right)$ is equal to 5.79E-11, $Var\left(\phi(\underline{x})\right) \sim 0$.

In this hypothetical example, in which Importance Sampling optimization is applied in the total available power of an area with 5 generators, the LOLP variance goes to zero when applying IS to the total available power distribution. This is possible because the available power of the 5 generators is a one-dimensional random variable that can assume 6 values corresponding to the number of generators operating (0, 3,6,9,12 or 15 MW).

Thereby, the minimal variance is obtained when importance sampling is applied to the total available power distribution function of each area, instead of to the distribution function of each generator individually. Or in other words, Importance Sampling Optimization should be considered in joint probability distribution of the system components instead of in each marginal distribution individually. Then, it leads to conclude that importance sampling techniques is more effective when used for univariate, rather than multivariate distributions.

Table 17 summarizes the results obtained in this chapter considering the hypothetical system with 5 generators $(G_1, G_2, G_3, G_4, G_5)$.

Table 17: Summary of the results - IS optimization in a hypothetical system

Generators with same failure rate		
	State Enumeration	Var (ϕ) = 2.58E-04
	Importance Sampling using samples of independent draws resulting in load shedding:	
	In each generator individually	Var (ϕ) = 1.2 E-07
E $(\boldsymbol{\phi})$ = 2.58E-04	In the total available power distribution function	Var (ϕ) = 5.01E-10 Var (ϕ) = ~ 0
Generators with different failure rate		
	State Enumeration	Var (ϕ) = 2.17 E-04
	Importance Sampling using samples of independent draws resulting in load shedding:	
	In each generator individually	Var (ϕ) = 7.61E-08
E $(\boldsymbol{\phi})$ = 2.17E-04	In the total available power distribution function	Var (ϕ) = 5.79E-11 Var (ϕ) ~ 0

From Table 17 is possible to obtain some conclusions:

- IS optimization is a technique that reduces the variance of an estimator and then, it's useful for power reliability assessment.
- Applying the IS optimization in the total available power distribution function is more efficient than applying it in each generator distribution, since estimator variance resulted from applying IS in the total available power is much smaller compared to variance value obtained applying IS to each generator individually. (1.2 E-7 vs. 5.01 E-10 for generators with same failure rate and 7.61 E-8 vs. 5.79 E-11 for generators with different failure rates).

- IS optimization is more powerful if applied in a one-dimensional distribution function.
- In the case of one-dimensional distribution function (using the available power capacity) variance estimator goes to zero, because the available capacities could be enumerated.

Nevertheless, electrical power systems have many components, and in real systems is not possible to enumerate all possible combinations of available power to apply IS. Thereby, a different approach is proposed to be used in this work. It allows application in real power systems and will be detailed in Chapter 5.

4.2 Markov Chain Monte Carlo

Markov Chain Monte Carlo (MCMC) is a method of state sampling in which each sample is obtained through a Markov Chain process.

In the context of reliability evaluation, this method allows sampling system states with load shedding, with each sample obtained by Markov Chain. In other words, this method will let moving in a state space transiting from one state to other.

Thereby the pre-requisite for obtaining load shedding samples by Markov Chain Monte Carlo in to know a priori one sample with the operative state of the systems' components that result in load shedding. Therefore, the purpose of MCMC applied to power system reliability is to get unserved energy scenarios [46] from one scenario of load shedding. In this work, the method of MCMC will be applied considering a multi-area reliability evaluation. In other words, it means that the samples obtained through MCMC are vectors containing the power available capacities in each area which result in load shedding scenarios.

The proposed application of MCMC in this work will be explained in detail in Chapter 5.

5 Proposed Methodology

This chapter aims to present the methodology that is proposed in this work. Before presenting all its details, some outlines of the methodology are indicated in the first section comparing it to some traditional methods and works that have already been published in the literature. Moreover, the first section of this chapter contains two flow charts indicating the steps to be followed while using the proposed methods. The first flowchart doesn't consider the renewable representation while the second one includes a representation for variable energy resources in the reliability evaluation assessment.

The main propose of these flowcharts is to present an overview of the methodologies and the techniques used. Each technique will be detailed from the second section to the end of this chapter.

5.1 Highlights of the Methodology

Reliability studies have always been important with systems electrification and the necessity to supply energy to consumers. Initially the studies of electrical systems consisted of the analysis of one electrical area, with a deterministic criteria of equipment failure. Progressively, reliability studies contemplate generation and transmission failure, as well as stochasticity in failure scenarios. However, since electrical systems are generally reliable, sampling scenarios in a system that result in load shedding is not trivial and often results in high computational effort. Thus, many samples obtained through the Standard Monte Carlo Simulation do not contribute to accumulating the reliability indexes.

The rareness of failure events is wide mentioned in the literature and to overcome this issue many variance reduction techniques have been proposed. Importance Sampling is a variance reduction technique generally used with the Cross Entropy (CE) technique to obtain the optimal sampling. However, CE-IS aims to determine an optimal density function for each component in the system through an interactive process. In other words, CE-IS focuses on determining a new forced outage rate for the elements in the system. Thereby, some elements that not even contribute to failure events are distorted through CE-IS. In [36] is mentioned that some components do not cause large impacts on the reliability indexes (mentioned as non-bottleneck components) and the authors propose identifying these elements among all in the system to not apply the IS.

Since the essential for the reliability analysis of power systems is the operating state of the system as a whole, instead of applying IS component by component, it is proposed in this work to use the MCMC, as it allows to obtain a set of samples that result in load shedding with low computational effort. MCMC is applied as a basis for the optimal importance sampling.

Thereby, considering that electrics systems are usual reliable and the reliability evaluation focus on studying rare events (scenarios that result in load shedding), MCMC technique is applied in this work for the sake of obtaining samples of vectors, that result in load shedding, containing the available power in each area. These vectors with events of infeasible states are used for the importance sampling technique that, in general words, aims to “tilt” the original distribution of the total available power of each area to increase the probability of sampling events that more contributes to failure in the system.

The fast insertion of renewables sources worldwide and their increasing participation in the energy matrix of many countries increased the importance to represent these sources in reliability studies. However, their daily profiles, time and spatial correlation with the load and among VER were translated into new challenges for their representation in reliability studies. Some works have been developed to contemplate intermittency and time and spatial correlation in reliability studies, such as [5] and [40], both considering a multi-area reliability assessment. In [5] is assumed that wind sources and the load follow specific distribution patterns, while [40] considers three profiles to represent the wind sources in the IEEE RTS-96 system. Similarly, this work proposes a multi-area reliability assessment, but stratification is proposed as a method to treat renewable sources and their characteristics. Stratification technique and its application in reliability studies will be explained in detail in this chapter.

Therefore, in order to reduce the computational effort in the study of the reliability of electrical power systems, it is proposed to apply Monte Carlo Markov Chain (MCMC), Importance sampling techniques, and Stratification for the reliability evaluation of a multi-area system.

The main highlights of the methodology that will be proposed in this work are:

- MCMC scheme to directly produce a sample of failure states, thus avoiding the CE iterative process.
- Multi-area reliability assessment based on optimal IS and MCMC.
- Stratification techniques to represent the fact that the underlying VRE stochastic process changes along the day, keeping the time and spatial correlation among VRE sources.

- Application in real systems, with great speed up if compared to the Standard Monte Carlo Simulation.

5.1.1 Overview of the Methodology – Without Renewable Treatment

The proposed methodology considers a multi area reliability assessment and then the first step consists in representing an equivalent of the system in multiple areas. Once defining the equivalent of the system in a multi-area modelling it is possible to obtain the density function corresponding to the total available power in each area of the system. Then, IS should be applied to distort these density function in order to increase the probability of sampling, for each area, a scalar of the total available power which contributes to failure events (load shedding events).

In order to obtain the optimal sampling, MCMC technique is applied to sample vectors with available power in each area that result in failure events. These vectors with events of infeasible states are used for the importance sampling technique. Thereby the vectors obtained through MCMC are considered in the IS, as illustrated in the green and blue boxes in the Figure 16. The IS Optimization and MCMC will be explained in detail in sections 5.2 and 5.3.

As previous mentioned, tilting each point of the original distribution is not efficient and then, in this work it is proposed to divide the original distribution in clusters (bins), and apply the importance sampling to each bin. The concept of bins and the advantages for its use will be further explored in section 5.2. Once the importance sampling is applied, a new probability value is determined for each bin so that the bins with an increase in their probabilities must be the ones that more contribute to not supplying the demand of energy. Considering this importance probability function, for each electrical area one bin is sampled and then, an available power generation must be sampled conditioned to the sampled bin. It results in a vector with dimension of the number of areas, composed by the available capacity of the generators in each area. A reliability evaluation is performed to analyze if this vector result in an event with load shedding, contributing to the LOLP index.

The process of sampling bins of the importance distribution and sampling a generation state from the corresponding truncated distribution of that bin is repeated until reaching the convergence. This process is indicated with pink boxes in the chart of Figure 16.

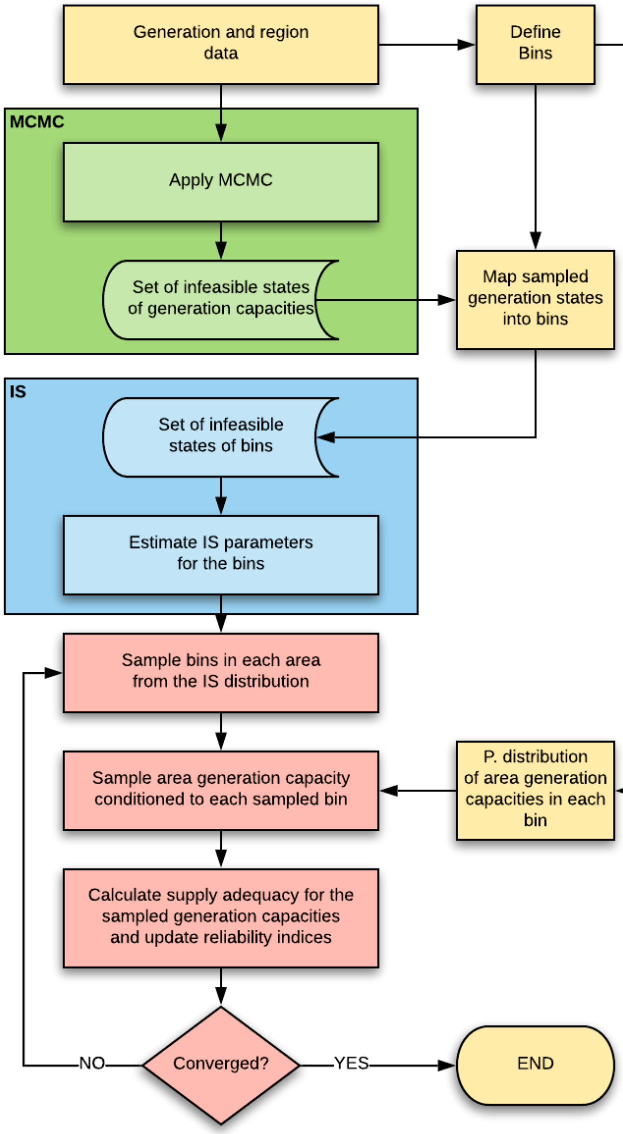


Figure 16: Methodology Flow Chart – not including the Renewable Treatment

5.1.2 Overview of the Methodology – Including the Renewable Treatment

As the integration of renewable generation grows in energy systems, it is important to consider and represent these intermittent sources in the analysis of the reliability of systems. However, unlike the thermal generation profile, intermittent renewable sources have non-stationary generation profiles, that is, for some hours of the day the generation profile can be completely different from other time of day. It is the case of solar sources, which generate only during the day and therefore, the failure of a solar generator contributes to the reliability indexes only if the generator is failed in moments in which there is solar irradiation. The breaking of a solar generator during the dawn would not impact on the reliability indexes. Thus, for the sake of considering the non-stationary behavior of intermittent sources in the reliability studies, it is suggested to apply stratification, which is a method that consists in dividing the state space into strata with similar characteristics and then analyze each of them individually.

The stratification method allows intermittent generation sources to be included in the reliability analysis. However, in having to analyze each stratum individually, it introduces the difficulty to determine the number of events of Monte Carlo to be drawn in each of the strata to calculate the reliability indexes. In other words, for a $\mathcal{N}$ set of draws, how many of $\mathcal{N}$ should be allocated to each stratum?

Ideally, the number of draws in each stratum should be higher in the stratum where the variance of LOLP was greater (it will be further demonstrated in this thesis). However, the occurrence and the probability of load shedding is just what we want to calculate. Then, to allocate the draws among the strata, it is suggested to determine an upper and lower bound for the LOLP. More draws are allocated in stratum with higher LOLP upper bound. The upper bound of the LOLP can be determined by the Hunter inequality that will be described in the section 5.5.3.

The calculation of the upper and lower limits of the LOLP, besides allowing to allocate draws among the strata, allows to eliminate strata in which the values of LOLP upper and lower bound are close. If the value of the upper limit of the LOLP of a stratum is very close to the lower bound it means that the LOLP real value of the stratum is "almost" known and then, there is no need to perform the MCMC and IS for this stratum. Thus, by eliminating strata, the computational effort and time in the calculation of the system reliability is reduced. This process of considering the calculated values of the LOLP lower and upper bounds to avoid carrying out MCMC and IS is illustrated in gray in the Figure 17.

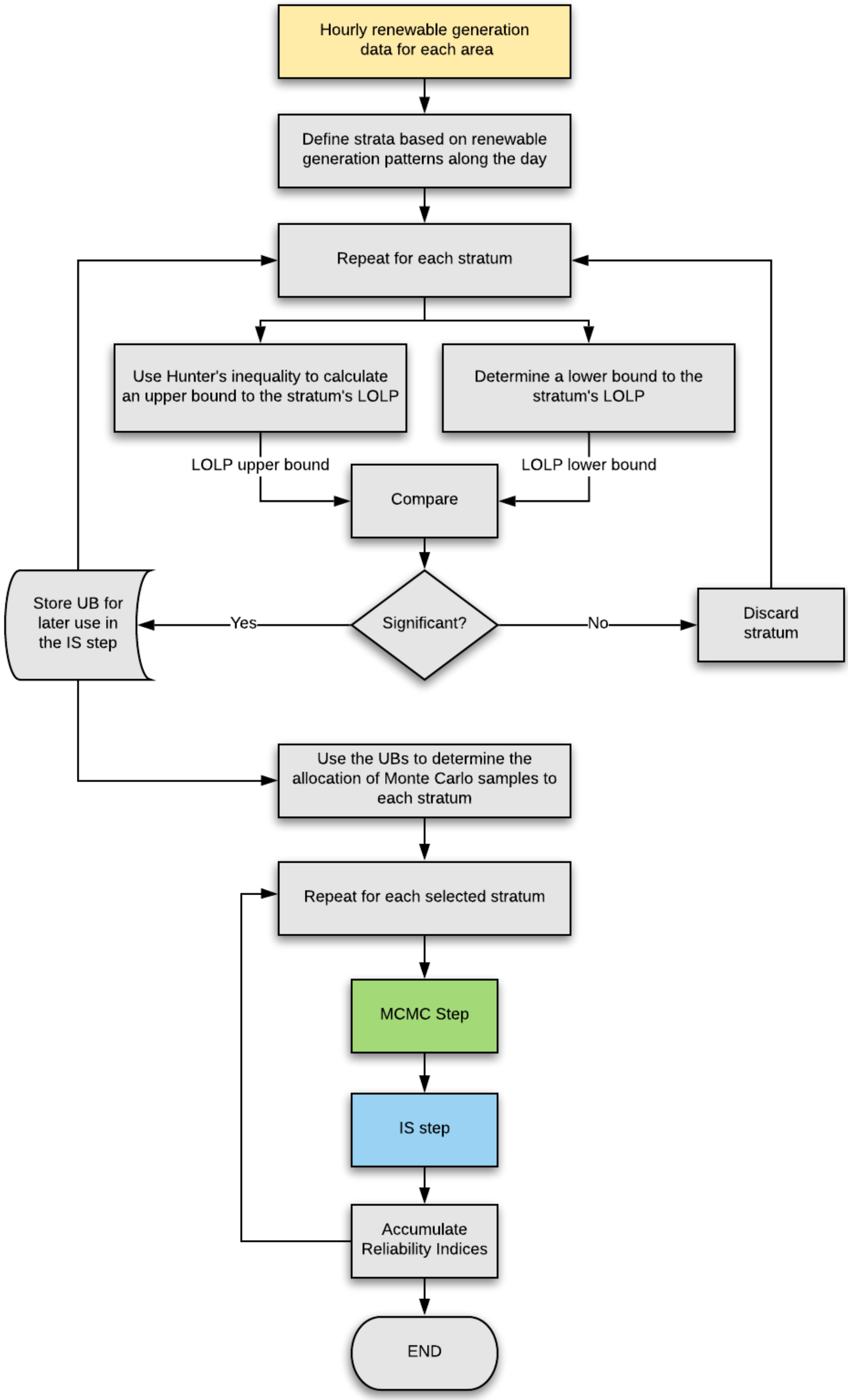


Figure 17: Methodology Flow Chart – including the Renewable Treatment

5.2 IS Optimization using "Bins"

The importance sampling optimization aims to reduce the variance of an estimator to decrease the effort and computational time in the analysis of the reliability of power systems by Monte Carlo Simulation. From the examples provided in the previous chapters the efficiency of this method is observed by the significant reduction in the variance of the estimator.

In the previous chapter, IS optimization was firstly introduced to skew the probability density function of each component in the electric network individually. However, electrical power systems are usually complex and with a considerable number of components and would take much time to distort the probability distribution of each component at once. In addition, reliability studies require scenarios that result in load shedding in the system which depends on the operating condition of all system components together. Thereby, a method that proposes to change the failure rate of the individual components would not be effective.

Then, another approach was proposed for IS optimization using the probability distribution function of the available power of each area in a system instead of the outage probability of the system's components. The procedure of using the probability distribution of the available power aims to reduce the number of dimensions in the optimization of the IS and, as consequence, increase the method efficiency. Note that the probability distribution of the available power is a univariate distribution and therefore IS optimization is more efficient since further reduces the variance of the estimator compared to the technique of individually distorting the probability of each component of the system.

Nonetheless, by applying IS optimization to the total available power probability distribution, the probability of each possible state of the system was individually distorted since they could be enumerated (in the example described in section 4.1.3). However, electrical systems in general have many components and then, distorting the probability of each individual state would be computationally exhaustive.

For the sake of increasing even more the performance of the IS optimization, in this work it is proposed discretizing the probability distribution of the total available power of each area. In other words, it corresponds to discretize the available power into classes or *bins*, whose original probability density function will be distorted to estimate an optimum distortion of each bin. The idea of creating bins or classes with truncated subsets of the original probability distribution is illustrated in Figure 18.

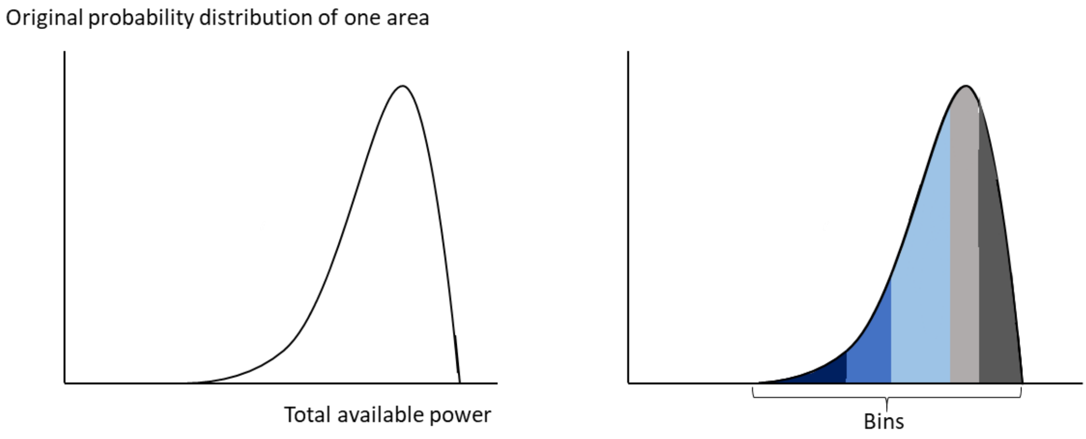


Figure 18: Defining bins in a distribution function.

5.2.1 Mathematical Formulation - Univariate case

Suppose now that $\{x^s, s = 1, \ldots, S\}$ is characterized by a set of *scalars* with the total generation availability capacity of a given area:

$$\overline{G}^s = \sum_i \overline{g}_i \times x_i^s, s = 1, \ldots, S \qquad (16)$$

Where $\overline{G}^s$ is the total generation availability capacity for the state s; $\overline{g}_i$ is the generation capacity of generator i and x_i^s the state of each generator i in state S.

Suppose these generation capabilities are aggregated into K clusters, or "bins." For example, each bin $k = 1, \ldots, K$ corresponds to a range of the total generation capacity. For K = 3, these intervals could be:

$k \ = \ 1 \Rightarrow (0; \frac{1}{3}\overline{G}]$

$k \ = 2 \Rightarrow (\frac{1}{3}\overline{G}; \frac{2}{3}\overline{G}]$

$k \ = 3 \Rightarrow (\frac{2}{3}\overline{G}; \overline{G}]$

Where $\overline{G} = \sum_i \overline{g}_i$

In this case, a sample $\{x^s, s = 1, \ldots, S\}$ of states can be written as independent draws of $k(s)$, where $k(s)$ is the bin to which each generating capacity belongs for state s. The goal of IS is to maximize the probability of drawing generation states in the bins that lead to supply failures. This is equivalent to determine the failure probability parameters of each bin $p^*_{k(s)}$ that maximize the probability of occurrence of the sample $\{x^s, s = 1, \ldots, S\}$.

This corresponds to the maximum likelihood estimate of the sample S. Since the draws are independent, the joint probability is given by:

$$\max_{p_k^*} \prod_s p_{k(s)}^* \qquad (17)$$

$$\text{Subjected to: } \textstyle\sum_k p_k^* = 1$$

Applying the logarithmic function:

$$\max_{p_k^*} \sum\nolimits_s \log\left(p_{k(s)}^*\right) \qquad (18)$$

$$\text{Subjected to: } \textstyle\sum_k p_k^* = 1$$

Rewriting the optimization function with the summation in function of the k bins:

$$\max_{p_k^*} \sum\nolimits_k S_k \log\left(p_{k(s)}^*\right) \qquad (19)$$

$$\text{Subjected to: } \textstyle\sum_k p_k^* = 1$$

Where S_k corresponds to the number of vectors whose scalar value corresponds to the bin k. ($\sum_k S_k = S$)

An optimization problem with one constraint can be solved using a Lagrange multiplier $\hat{\mu}$. Thereby, the optimization problem can be rewritten as:

$$\max_{p_k^*} \sum\nolimits_k S_k \log\left(p_{k(s)}^*\right) - \hat{\mu} \times \sum\nolimits_k p_k^* \qquad (20)$$

The optimal values can be obtained by deriving and equating to zero each variable p_k^*:

$$\frac{S_k}{p_k^*} - \hat{\mu} = 0 \quad k = 1, \dots, K$$

Then:

$$p_k^* = \frac{S_k}{\hat{\mu}} \qquad (21)$$

The Lagrange multiplier $\hat{\mu}$ is then determined by:

$$\sum\nolimits_k p_k^* = 1 \;\Rightarrow\; \sum\nolimits_k \frac{S_k}{\hat{\mu}} = 1 \;\Rightarrow\; \sum\nolimits_k S_k = \hat{\mu} \;\Rightarrow\; \hat{\mu} = S$$

Replacing the Lagrange multiplier $\hat{\mu}$ in the previous equation:

$$p_k^* = \frac{S_k}{S} \quad k = 1, \dots, K \qquad (22)$$

Therefore, the optimal probability of each bin k in the IS is given by the proportion of draws of each bin k in the total number of draws S that result in load shedding.

In this work the concept of bins will be used with the propose of applying importance sampling to each bin to obtain a new probability distribution of the available power of each bin.

In summary, Importance Sampling is a variance reduction technique that is applied in this work to reduce the variance of the LOLP estimator for energy supply reliability evaluation. In order to reduce the computational effort to distort the probability distribution of the available power that is a continuous distribution which contains several values, it is suggested to divide the original probability distribution into bins. Each bin contains a truncated subset of the original distribution and then the Importance Sampling technique is applied to each bin.

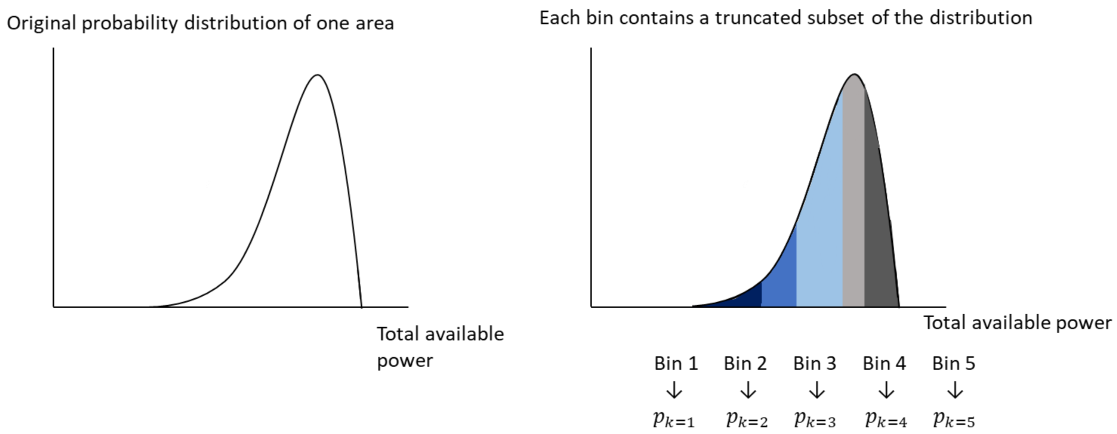


Figure 19: Bins containing a subset of the truncated original distribution

From Equation (22) is possible to observe that the procedure to obtain the distorted marginal probability distribution of each bin requires independent samples that result in load shedding. Once the samples are known, the distortion of each bin is obtained by the frequency of each bin in the samples that resulted in a load shedding in the system (as shown in Equation(22)). At this point the distorted probability of each bin p_k^* is determined.

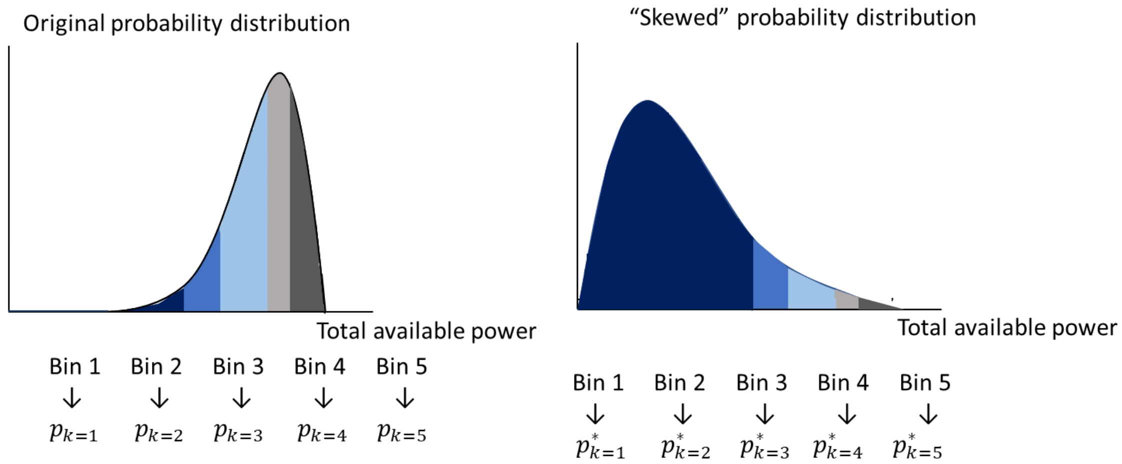


Figure 20: Importance probability of each bin

From Figure 20 is possible to observe that IS aims to increase the probability of sampling in bins in which the total available capacity is smaller and, then, it is more likely to sample load shedding scenarios.

Once p_k^* of bins are determined by importance sampling optimization, Monte Carlo simulation is applied to sample bins from the new distribution function (p_k^*).

For this area, sample one bin (k) and then sample a generation capacity state from the corresponding truncated distribution of that bin (x_s^k). Let $p(x^k)$ be the original probability distribution of the sampled bins sampled and $p^*(x^k)$ the corresponding importance probability. The estimator expected value can be determined by:

$$\overline{E}\left(\phi(\underline{x})\right) = \frac{1}{S}\sum_{s=1}^{S}\frac{p(x^k)}{p^*(x^k)}\,\phi(x_s^k) \tag{23}$$

Note that the importance sampling is applied to the bins, i.e., only the distribution probabilities of the bins is "tilted". However, the calculation of the expected values of the reliability indexes is done based on the generation capacity whose probability is not distorted or "tilted".

Hence, the IS optimization using bins can be summarized by the following steps:

1. Divide the probability distribution of the available generation capacity into k clusters or bins. Each bin will have a truncated subset of the distribution.
2. Apply the Importance Sampling Optimization technique to change and "tilt" the original probability of each bin. (IS is applied to the bins)
3. Sample bins considering the importance probabilities determined in step 2.

4. Given a sampled bin of step 3., sample a generation capacity state from the corresponding truncated distribution of that bin.
5. Determine the estimator expected value using Equation (23).

To execute step 2, an IS optimization technique is needed. In this section it was assumed that the set of infeasible events (S) was previously known. However, it's not known a priori. Thereby, for the sake of determining these infeasible events a *Markov Chan Monte Carlo* (MCMC) simulation is applied.

5.3 MCMC Method

In the previous section, the IS optimization technique was introduced for the sake of applying this variance reduction technique to reduce the computational effort in the power reliability assessment. Nevertheless, the proposed IS optimization method requires knowing load shedding scenarios to "tilt" the bin's probabilities.

Notwithstanding, one of the difficulties of the reliability assessment is to get these load shedding scenarios. So, this section aims to introduce the Markov Chain Monte Carlo applied in this work for the reliability assessment to guarantee obtaining these load shedding scenarios.

The pre-requisite for obtaining load shedding samples by Markov Chain Monte Carlo is to know a priori one sample with the operative state of the systems' components that result in load shedding. From the event that result in load shedding, the principle of MCMC is to gradually increase the power availability of one area, keeping constant the value of the available capacity in the other areas, until reaching the border from which there is no more load shedding. Because the $2^N - 1$ (N is the number of areas), Hoffman-Gale feasibility cuts are known and the polyhedron corresponding to infeasible states can be defined.

The MCMC iteration method presupposes knowing one load shedding state from which is possible to obtain other samples that result in load shedding scenarios. In other words, MCMC sampling is applied to obtain new events in the infeasibility polyhedron.

The MCMC sampling can be performed solving a linear optimization problem varying the available power of each area individually. So, for one load shedding state and supposing at initially a specific load value, a vector with N components corresponding to the available power in each area is known ($[A_1^0, A_2^0, \ldots, A_N^0]$). Fix all components in this vector, except for one area, move in the direction of increasing the available power of this area (keeping the others constant) until one boundary is obtained. Then, sample one value in the marginal truncated density function of the available power of this area obtaining a new vector $[A_1^1, A_2^0, \dots, A_N^0]$. This power capacity and the available powers of the other areas that were kept constant produce a load shedding vector. This same process is repeated until the boundary of each area is reached.

An example of this process for a two-area system is illustrated in the Figure 21. Initially suppose the MCMC procedure considering a specific load value (L). Suppose two areas in a system ($[A_1, A_2]$), each one with an available power density function obtained by the convolution of the available power of the generators in each area. Consider a demand L and the initial load shedding event with 1000, 800 MW ([1000,800]) as the power availability in each of the two areas, respectively. Consider that the available power of each area follows a probability density function with the maximum available power equal to 4000 MW and 6000 MW.

For the sake of applying MCMC method, move on the direction of increasing the available capacity of one area, but don't change the available power of the other areas. For instance, fix the available power of A_1 in the original value (1000 MW), but increase the available power at area 2 (A_2) until reaching the "non-feasible" boundary region, i.e., from which the demand L is met.

It is worth emphasizing that reducing the available power would only increase unsupplied demand. Thus, during the MCMC process, the boundary between served and unserved energy is always reached increasing an area available power. Suppose that the demand L in this example is supplied when the generation capacity in area 2 is $5100\ MW$. A new truncated density function of A_2 is gotten with the available power varying from 0 until 5100 MW. Then a value of A_2^1 is then drawn from the truncated distribution (with $A_2^{Max} = 5100\ MW$).

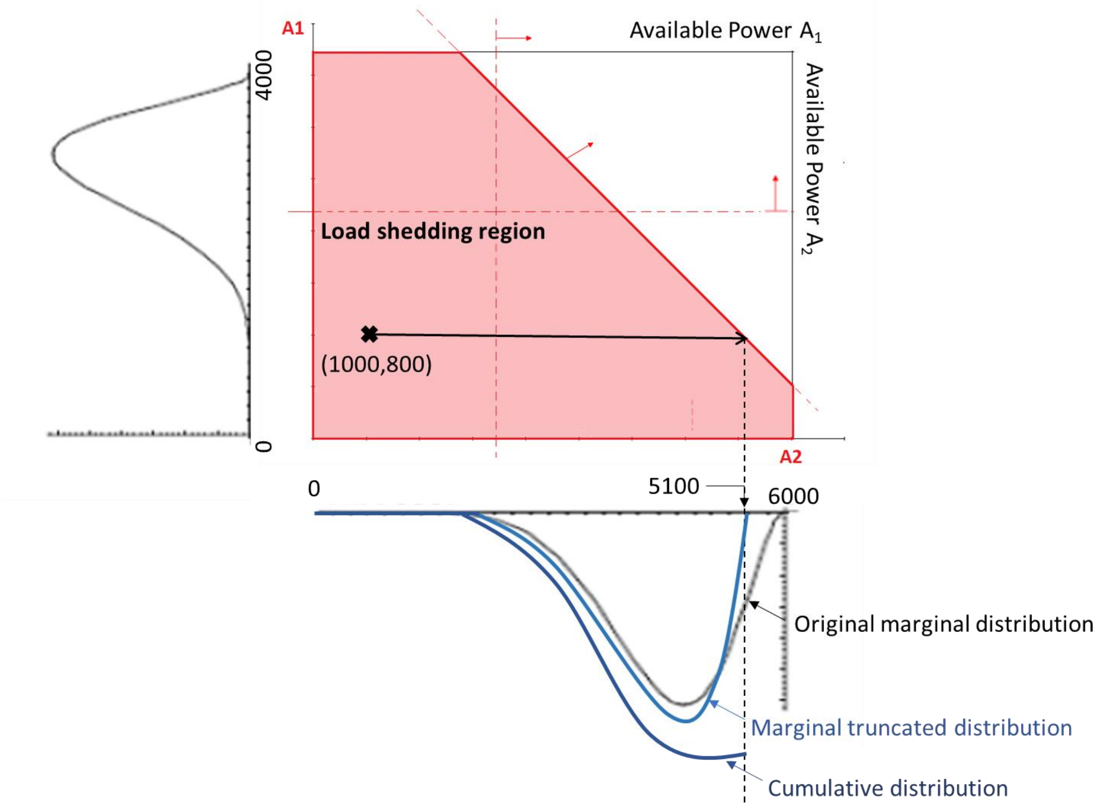


Figure 21: MCMC example

The same process can be repeated to obtain one value from the truncated distribution of A_1, keeping A_2^1. Then, $[A_1^1, A_2^1]$ correspond to a new load-shedding scenario. Repeating this process, at the end of MCMC method several load shedding scenarios are obtained.

For the sake of applying the importance sampling optimization technique, which was proposed in section 5.2, it is necessary to obtain independent samples of load shedding. Therefore, it is suggested to discard each N samples obtained by MCMC (N is the number of systems' areas), to obtain the set of "effective" samples that result in unserved energy scenarios. For instance, if 30,000 load-shedding scenarios are obtained by MCMC for a system with five areas, 6,000 of them would correspond to the effective scenarios consider for the importance sampling.

It is noteworthy that each load shedding state from MCMC is gotten from solving N optimization problems. In the given example, it is required to solve 2 optimization problems to obtain a sample from MCMC method ($[A_1^1, A_2^1]$) , since it's a two-area system. It might lead someone to think that MCMC would not be efficient. Nevertheless, electrical systems are usually divided into a small number of areas and then the MCMC methodology is promising in the context of multiarea reliability assessment.

So, the MCMC technique lets generating load shedding scenarios that can be used in the importance sampling optimization. As explained in the previous chapter, the IS technique aims to “tilt” the original probability distribution to increase the probability of sampling a set of infeasible states of generation capacity. As indicated in the previous chapter, instead of “tilting” point-to-point from the original distribution, bins with a truncated subset of the original distribution are created. Thereby, MCMC and IS are applied in those bins.

Thus, the vectors with the values of the available power sampled by MCMC must be associated to the bins whose probabilities will be distorted by IS according to the relative frequency of draws in each bin as indicated in Equation (23).

5.3.1 MCMC considering the load profile

Traditionally in reliability studies it was assumed that most load shedding events occurred in periods of heavy demand. Thereby, reliability evaluation was commonly considered for hours with highest demand. However, with the increase of intermittent renewable sources in the systems’ energy matrix, it’s even more likely that supply failure occurs at times of medium or light load, depending on the contribution of renewable sources in the system. So, it’s important to consider different load levels for the reliability studies, since supply failure scenarios don’t necessarily occur for heavy load conditions.

In the example of the previous section, MCMC method was illustrated considering a fixed demand value (L) to be met. So, all vectors generated by MCMC resulted in load shedding for this specific demand. Nevertheless, MCMC can be extended to consider a demand profile during the MCMC simulation.

In order to consider the demand profile in the MCMC instead of only one value (typically the maximum demand), the same process described in the previous section is considered, except for adding one more dimension in the MCMC that corresponds to the dimension of the *total demand of the system*. In this work, it’s supposed that the demand profile of each area individually is proportional to the demand profile of the system. This premise lets considering just one dimension for the demand in the MCMC and applying a disaggregating factor (proportional to the demand of each area) it’s possible to verify the load shedding in each area. In other words, adopting this approximation that the demand in each area follows the same profile as the total system demand, that is, it is a fixed proportion of the total demand, total demand is added as dimension N+1 of the infeasibility polyhedron. In order to avoid serial correlation of sampled states N+1 following states are discarded, and the next event is used to update the relative frequency of each bin according to (22).

The difference in the MCMC method when considering the demand is that unlike the generation capacity dimension, the feasibility boundary is reached moving towards the direction of decreasing the total demand.

So, from one initial event that result in load shedding, gradually decrease the total demand of the system until reaching the feasible boundary. Similarly, to the procedure described when just considering one demand value, generate a sample from the truncated density function of the total demand of the system. Then, from this point, gradually increase the power availability of one area, keeping constant the value of the available capacity in the other areas and the demand value, until reaching the border from which there is no more load shedding. The new event resulting in load shedding is completed with the available powers values of all areas and the new infeasible demand value.

5.4 Monte Carlo Simulation

As indicated in the flowchart presented in Figure 16, after the MCMC sampling process and the Importance Sampling Optimization in the bins of each area in the system, the Monte Carlo Simulation is performed. The samples in Monte Carlo Simulation are drawn from the importance distributions and for each sample (s) the LOLP index is determined by Equation (23) for a univariate case. i.e., case with just one are. For system with N areas, the LOLP index is calculated as indicated in Equation (24).

$$\overline{E}\left(\phi(\underline{x})\right) = \frac{1}{S}\sum_{s=1}^{S}\frac{\prod_{n=1}^{N+1} P_n\,(k_n^s)}{\prod_{n=1}^{N+1} P_n^*(k_n^s)}\,\Phi(x_1(k_1^s), \dots, x_{N+1}(k_{N+1}^s)) \tag{24}$$

Where k_n^s is the bin of component *n*, $P_n({k_n}^s)/P_n^*\ ({k_n}^s)$is its likelihood ratio (LR), and Φ is the indicator function evaluated at state $(x_1(k_1^s), \dots, x_{N+1}(k_{N+1}^s))$. It should be noted that the $x_n(k_n^s)$ value of each component is sampled from the corresponding original distribution, limited to the range of the bin k_n^s.

5.5 Stratification - Variable Energy Resources in Reliability Studies

In the previous sections the concepts of Importance Sampling Optimization and Markov Chain Monte Carlo applied in this work were introduced. However, they didn't consider any treatment for representing renewable sources in reliability studies.

Thereby, this section presents some characteristics of renewable sources that require a special treatment for their representation, a description of a methodology to include VER in the reliability analysis evaluation and it also extends the IS+MCMC methodology to represent VERs in reliability assessment.

5.5.1 Renewable Sources Representation

The fast insertion of VER, such as wind and solar generation worldwide has made reliability studies more relevant but also more complex. It's necessary to represent VER variability, unpredictability and their non-dispatchable characteristics. Moreover, it's important to consider possible time and spatial correlation among VER (portfolio effect). Furthermore, renewable sources require a more detailed and smaller simulation time step to represent their daily profiles (such as hourly resolution simulations). In addition, in some cases, there is also a complex relationship between renewable generation and loads.

The Figure 22 illustrates the average generation profile of solar and wind sources in Chile for the month of December. It's possible to observe a time correlation between these renewable sources since the solar sources generate during periods with solar irradiation while the wind sources generate more during the night.

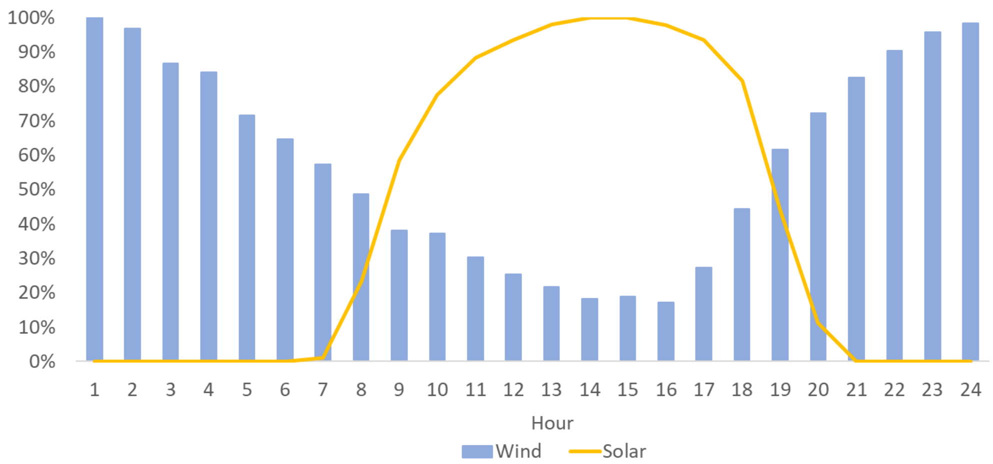


Figure 22: Average generation profile in Chile in December.

The thermal generators' operating state impact the reliability indexes of the system in the same way regardless of the time instant under analysis. However, the same does not happen, for example, in the case of solar plants that produce energy, contributing to demand supply, typically between 9 am and 6 pm. Although the solar failure rate is the same throughout the day, its total available power varies during the day.

Thereby, if the operating state of a solar power plant in a Monte Carlo draw is "failure" it does not impact the adequacy analysis of the system if the reliability evaluation occurs for the dawn period because the total available power of solar plants at dawn is zero. However, if the generator failure occurs at noon, it will impact the reliability indexes. Therefore, the solar plants have a non-stationary available power and require a different treatment in the study of reliability.

So, renewable sources representation, such as the solar generation, require a special treatment due to their stochastic process throughout the day. Therefore, in this work the *stratification* technique is proposed to divide the state space of renewable sources into subgroups. These subgroups must be composed by consecutive hours with a similar available power distribution to let considering each subgroup individually and as a stationary available power distribution function. These subgroups are called strata (or stratum in the singular).

For the sake of illustrating the process of creating strata, consider the total available power of a solar plant in a daily cycle as indicated in the Figure 23. The stratification process aims to divide the total power into strata so that the stochastic process of the available power of the solar in the same stratum is as close as possible.

So, for example, it could be divided in four strata according to the total power throughout the day: during the night, sun rise, during the day and during the sun set, indicated respectively with numbers from one to four, in the Figure 23.

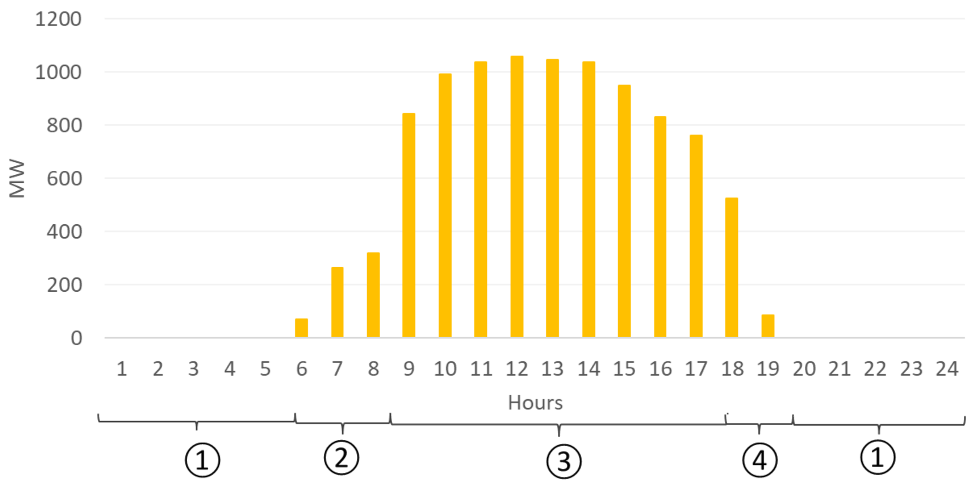


Figure 23: Four strata example based on solar plant available power

Applying the stratification method dividing the original profile into four different strata could led to apply the MCMC and IS techniques for each one of these strata individually. Then, in other words, the process explained in the previous chapter of defining bins, applying MCMC and then the IS optimization in the bins to then get samples from the importance probability to determine LOLP expected value should be repeated for each stratum. The system LOLP is then calculated as a function of the LOLP of each stratum weighted by the probability of each occurrence.

On one hand, the stratification method allows intermittent generation sources to be included in the reliability analysis. On the other hand, the creation of strata introduces new issues. One of them, for example, is how to define the number of events to be drawn in each stratum when applying the Monte Carlo Method. Or in other words, for a $\mathcal{N}$ set of Monte Carlo draws, how many samples should be drawn from each stratum to calculate the LOLP of the system. Another issue is how the computational simulation time behaves when having to apply the MCMC and IS method for each stratum individually. These points will be discussed in the following sessions.

5.5.2 Number of samples to be drawn in each stratum

The stratification technique can be applied to divide renewable generations into groups that have similar stochastic processes. Once the strata are defined, the MCMC and IS method would be applied individually for each stratum. Then, samples obtained from the distorted probability distribution of the bins of each stratum are used to calculate, by Monte Carlo, the value of the LOLP estimator of the system.

Intuitively, the number of draws to be drawn in each stratum should vary depending on the LOLP in each stratum and on the probability of each stratum. It is then proved that the number of samples in each stratum should be proportional to the square root of the LOLP variance in each stratum (or the standard deviation of each stratum) and the probability of each stratum. For simplicity, at first consider that there are S strata, each one with the same probability, and that the LOLP value in each stratum is already known. Then, the demonstration is extended for strata with different probabilities.

5.5.2.1 Strata with same probability

For this demonstration consider S strata, each one with the same probability. The main goal is to allocate a set of $\mathcal{N}$ samples among the strata for the sake of minimizing the variance of the expected value of the system LOLP. This problem can be mathematically written by:

$$Min\ Var\left(\overline{E}\left(\phi(\underline{x})\right)\right) = \ Min\ \left(\frac{1}{\mathcal{N}}\,Var\left(E\left(\phi(\underline{x})\right)\right)\right) \tag{25}$$

Where $Var\left(\overline{E}\left(\phi(\underline{x})\right)\right)$ corresponds to the variance of the LOLP estimator, while $E\left(\phi(\underline{x})\right)$ is the LOLP value.

To contemplate the strata and considering that the LOLP estimator value of each of them is known, this objective function can be rewritten, and the mathematical formulation is:

$$Min \sum_{h=1}^{H}\left(\frac{1}{\mathcal{N}_h}\right)Var\ (\phi_h) \tag{26}$$

Subject to:

$$\sum_{h=1}^{H}\mathcal{N}_h = \mathcal{N} \tag{27}$$

$$\mathcal{N}_{h_i} > 0 \qquad \forall\ h \in [1, H] \tag{28}$$

Where ϕ_h corresponds to the LOLP estimator value of the stratum h, $\mathcal{N}_h$ is the number of draws in stratum h, H is the number of strata.

This optimization problem (Equation (26)) can be rewritten, considering μ as the Lagrange multiplier.

$$Min \left(\sum_{h=1}^{H}\left(\frac{1}{\mathcal{N}_h}\right)Var\ (\phi_h)\ \right) + \mu\sum_{h=1}^{H}\mathcal{N}_h \tag{29}$$

This optimization problem is solved deriving each term in relation to $\mathcal{N}_h$ and equaling to zero, obtaining that:

$$\mathcal{N}_h = \frac{\sqrt{Var\ (\phi_h)}}{\sqrt{\mu}} \tag{30}$$

Since the loss of load supply is an indicator function, i.e., a random variable that assumes only two values: zero or one, the LOLP variance of each stratum h can be also written as $Var\ (\phi_h) =\ \phi_h(1 - \phi_h)$. Since the probability of load shedding in electrical systems is usually small, the variance of the LOLP in each stratum can be approximated for ϕ_h. Thereby, the Equation (30) can be rewritten as:

$$\mathcal{N}_h = \frac{\sqrt{\phi_h}}{\sqrt{\mu}} \tag{31}$$

5.5.2.2 Strata with different probabilities

The previous section demonstrated that for a $\mathcal{N}$ set of samples, the optimal way to divide these sample among strata with the same probability of occurrence is proportional to the LOLP variance of each stratum.

Nevertheless, these strata may not have the same probabilities. Consider, for instance, the sample example provided in section 5.5.1, in which the strata definition resulted from the solar renewable representation. In this case there were four strata, each one with a duration corresponding to groups of hours of the day: hours during the night, sun rise, hours during the day and during the sun set. Thereby, in this case, what should be the optimal allocation of $\mathcal{N}$ draws among strata, since the probability of each stratum is different from each other?

In case of strata with different probabilities, the LOLP expected value can be determined as:

$$\bar{E}(\phi(\underline{x})) = \bar{\Phi} = \sum_{h=1}^{H} p_h \, \bar{E}\left(\phi_h(\underline{x})\right) \tag{32}$$

Where H is the number of strata, p_h is the probability of each stratum h and $\bar{E}\left(\phi_h(\underline{x})\right)$ the LOLP expected value of the stratum h.

This equation can be written as:

$$\bar{\Phi} = \sum_{h=1}^{H} p_h \, \frac{1}{\mathcal{N}_h} \sum_{j=1}^{N_h} \phi_h(x_{ij}) \tag{33}$$

The variance of the stratified sampling estimator is:

$$Var\,(\bar{\Phi}) = \sum_{h=1}^{H} \frac{p_h^2}{\mathcal{N}_h} \, Var(\phi_h) \tag{34}$$

Consider that a maximum number of $\mathcal{N}$ can be collected, i.e., $\sum_i^S \mathcal{N}_h = \mathcal{N}$, the optimal value of $\mathcal{N}_h$ that gives the minimal variance in each stratum is represented solved as:

$$Min \; \sum_{h=1}^{H} \frac{p_h^2}{\mathcal{N}_h} \, Var(\phi_h) \tag{35}$$

Subject to:

$$\sum_{h}^{H} \mathcal{N}_h = \mathcal{N} \tag{36}$$

$$\mathcal{N}_{s_i} > 0 \qquad \forall\ s_i \epsilon\ [1, S] \tag{37}$$

This optimization problem (Equation (35)) can be rewritten, considering μ’ as the Lagrange multiplier.

$$Min\left(\sum_{h=1}^{H} \frac{p_h^2}{\mathcal{N}_h}\ Var(\phi_h) + \mu' \sum_{h}^{H} \mathcal{N}_h\right) \tag{38}$$

This optimization problem is solved deriving each term in relation to N_h and equaling to zero, obtaining that:

$$\mathcal{N}_h = p_h \frac{\sqrt{Var\ (\phi_h)}}{\sqrt{\mu}} \tag{39}$$

However, the LOLP estimator value is one of the indices that one wishes to obtain in the reliability study of electrical power systems and, therefore, it is not known a priori. Thereby, to allocate the Monte Carlo draws among the strata it is suggested to use an estimation for the LOLP estimator value. The methodology for estimating limits (upper and lower bounds) for the LOLP is presented in the following section.

5.5.3 Bounds for the LOLP estimator

As shown in previous chapters, once obtaining the importance probability of the available power capacity in each stratum, by MCMC and IS (method described in Figure 17) the Monte Carlo method is applied to determine the LOLP estimator value from samples obtained from the importance probability of bins.

In order to apply Monte Carlo, it is necessary to define the number of samples to be drawn in each stratum. As shown in the previous section, the optimal allocation of these $\mathcal{N}$ samples among strata would be proportional to the standard deviation of the LOLP estimator of each stratum. However, the value of the LOLP estimator is not known and therefore, an estimate must be considered as an estimator of LOLP value.

Thus, this section aims to:

i. Present the methodology used in this work to define a lower bound and upper limit for the LOLP value in each stratum that can be used as a "metric" to allocate a set of samples among the strata;

ii. Indicate how estimating bounds for the LOLP estimator value can be useful for reducing the computational effort in the reliability study.

5.5.3.1 Accessing upper bounds for the LOLP estimator

In Chapter 3, it's indicated some approaches for analyzing the continuity of the energy supply in a power system. In this work it's considered the max flow-min cut theorem (described in the section 3.1.2) due to some advantages that will be evidenced in this section.

As described in Chapter 3, a multi area system can be represented using graphs whose cuts can be defined a priori, that is, they can be enumerated according to the number of areas (or nodes in the graph). A supply failure scenario just occurs if at least one feasibility cut is violated. In other words, to exist a supply failure scenario and a non-zero LOLP value, it is necessary that at least one feasibility cut be violated. It means that the LOLP estimator value can be estimated according to the probability of violating feasibility cuts.

Therefore, for a set of generic network flow constraints, such as those indicated below, the goal is to determine the probability of these constrains being violated.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots a_{1n}x_n \geq b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots a_{2n}x_n \geq b_2$$
$$\vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots a_{nn}x_n \geq b_n$$

Where a_{ij} assumes value 0 or 1 in a network flow and b_i is a constant.

One of the more general methods for obtaining upper and lower bounds for the probability of union of finite events is the Bonferroni equation, given by:

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{1\leq i\leq j\leq n} P(A_i \cap A_j) + \ldots \sum_{1\leq i_1\leq i_2\leq n} P(A_{i_1} \cap A_{i_2} \cap \ldots A_{i_n}) \quad (40)$$

Where A_i can be understood as the probability of a feasibility cut i be violated and $P(A_i \cap A_j)$ is the probability that both feasibility cuts A_i and A_j are violated at the same time.

Truncating terms of the previous equation allows obtaining upper or lower limits. By truncating the terms of odd order, upper limits are obtained. Otherwise, a lower limit is obtained truncating the even terms of the probability of the union of events. Thereby:

$$P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{k} (-1)^{j-1} S_j \qquad \text{For odd values of k = \{1, 2, ..., n\}} \quad (41)$$

$$P\left(\bigcup_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{k}(-1)^{j-1}\mathcal{S}_j \qquad \text{For even values of k = \{1, 2, ..., n\}} \tag{42}$$

Where:

$$\mathcal{S}_j = \sum_{1 \leq i_1 \leq i_j \leq n} P(A_{i_1} \cap A_{i_2} \cap \dots A_{i_j}) \tag{43}$$

According to [43], the most well-known improvement from the bounds of Bonferroni equation is the Hunter's inequality that states that an upper bound can be determined considering:

$$P\left(\bigcup_{i=1}^{n} A_i\right) \leq \mathcal{S}_1 - max \sum_{(i,j)\in\tau^*} P(A_i \cap \mathrm{A_j}) \tag{44}$$

Where $\mathcal{S}_1$ in this context, is the sum of the probability of each one of the feasibility cuts of a graph be violated individually, i.e., $\mathcal{S}_1 = \sum_{i=1}^{n} P(A_i)$. τ^* is the spanning tree of a graph which maximizes the value of $P(A_i \cap \mathrm{A_j})$ of this inequality. $P(A_i \cap \mathrm{A_j})$ represents in a graph the weight of the edge between the nodes A_i and $\mathrm{A_j}$.

It's possible to observe from inequality (44) that Hunter's upper bound basically resembles the same structure of the Bonferroni inequality truncated at the second degree, except for the fact that the presented Bonferroni inequalities are formed by the linear combination of events, while the Hunter's inequality consider in the second degree the maximum spanning tree.

In other words, to calculate the second term of the Hunter's inequality (44) it's considered a graph with the nodes representing the feasibility cuts and the arcs the probability of violating each two feasibility cuts. The second term of the upper limit for the estimation of the expected value of LOLP is obtained considering the largest value of the generating tree of the graph.

Intuitively, the algorithm for obtaining the maximum spanning tree allows eliminating redundant arcs that if used would result in double counting.

In order to facilitate the understanding of the maximum generating spanning tree concept, consider the graph with 4 nodes in Figure 24. In this graph, the nodes correspond to feasibility cuts and the arcs p_{ij} the probability of disrupting feasibility cuts i and j at the same time. If feasibility cuts 1 and 2, and 1 and 4 are violated, cuts 2 and 4 are certainly violated and then, should not be considered in the second degree of the Hunter inequality.

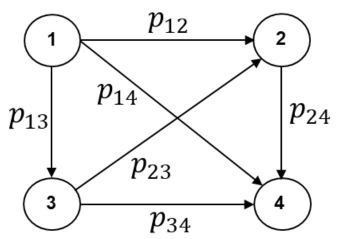


Figure 24: Graph with four nodes, each one corresponding to feasibility cuts

Thereby, the main goal is to "open" the graph in order to obtain the maximum tree in the graph, i.e., the tree whose capacity of the arcs sums the highest value.

- **<u>Determining the Hunter Inequality terms</u>**

Ideally, the number of drawings in each stratum should be proportional to the standard deviation of the expected LOLP value. However, since this value is what we want to calculate, it is not known a priori. Then, it is suggested to estimate an upper limit and lower limit for the LOLP estimator.

The upper limit can be determined by Hunter inequality. The Hunter limit value is obtained by the difference between the sum of the probability of violating each cut individually and the sum of certain terms of order two, computed as the weight of the maximum generating tree of the graph (Equation (44)).

This section aims to discuss the procedure to calculate the probabilities of violating the feasibility cuts and then, estimate the LOLP upper limit.

As shown in chapter 4, the number of cuts in a graph is given by 2^N, where 2^{N-1} comprises the feasibility cuts, where the left-hand side (LHS) of the cut is composed of the total available power of the areas in the cut, while the right-hand side (RHS) is composed by the total demand subtracted from the capacity of the interconnections in the cut. For a two-area system, the three feasibility cuts are:

$$f_{S_o,A_1} + f_{S_0,A_2} \geq f_{A_1,T} + f_{A_2,T} \quad (45)$$

$$f_{S_o,A_2} \geq f_{A_2,T} - f_{A_1,A_2} \quad (46)$$

$$f_{S_o,A_1} \geq f_{A_1,T} - f_{A_2,A_1} \quad (47)$$

The probability of each cut be violated individually can be calculated straight forwarded. For a feasibility cut with just one area on the left-hand side, the probability of this feasibility cut be violated is the integral of the available power distribution of the corresponding values under the value in right-hand side. So, considering equations (46) and (47), the probability of this cuts being violated correspond to the areas indicated in Figure 25:

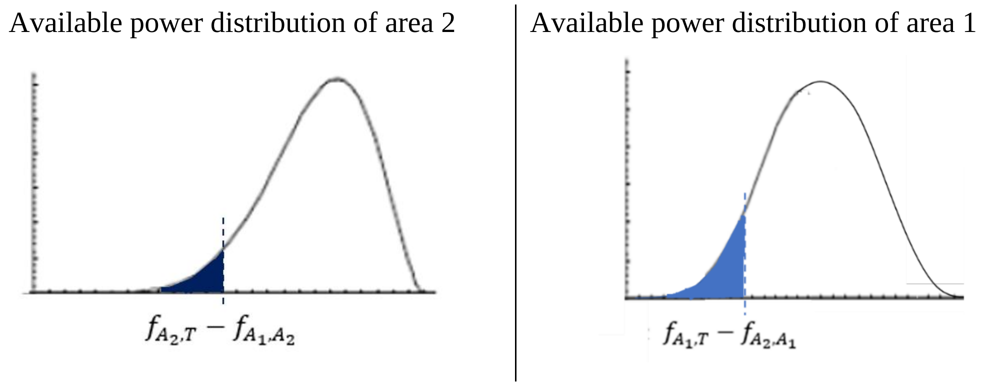


Figure 25: Areas corresponding to the probability of violating feasibility cuts

For feasibility cuts with more than one area in a cut (Equation (45)) , the probability of violating this cut is determined calculating the integral of the distribution obtained by numerical convolution of the available power distributions of each area for the values below the right hand side of the corresponding cut.

However, in order to determine the Hunter limit, you must also calculate the probability of two cuts being violated at the same time (second term of Equation (44)). In cases where the areas in the left-hand side of the two cuts are different, this probability is given by the product of the probability of each cut being individually violated. For instance, the probability of the two feasibility cuts represented in equations (46) and (47) be violated at the same time is given by the product of each of these feasibility cuts be violated individually. Nevertheless, in cases with the same areas in two cuts (Equations (45) and (46) or (45) and (47)), its more complicated to calculate the probability of two feasibility cuts be violated at the same time because it is a bivariate distribution. So, obtaining this probability would imply in determining the joint probability function and integrating in the defined region in the interval corresponding to the right-hand side of the two restrictions.

Calculating the joint distribution can be prevented since an analytical calculation simplifies to the calculation of first order terms. In order to do so, one must add the areas that are simultaneously in the two feasibility cuts, by numerical convolution, and determine for each of the cuts the probability of being violated. Then, the probability of violating both cuts at the same time is defined as the minimum value between the probability of each cut being violated.

The following example illustrates the process of defining the probabilities of violating feasibility cuts. Consider a three-area system, illustrated in Figure 26. The following inequalities represents the feasibility cuts.

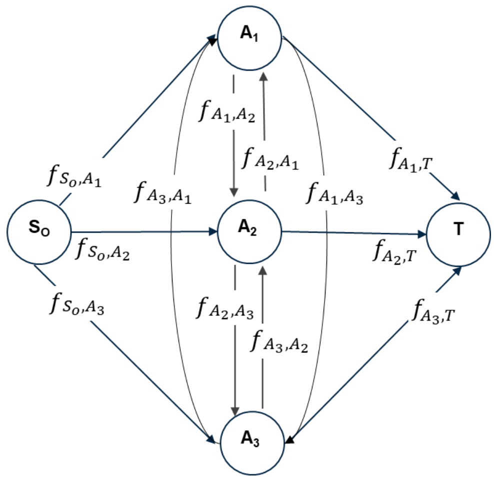


Figure 26: Three-area system

f_{S_o,A_1}: available power in area 1

f_{S_o,A_2}: available power in area 2

f_{S_o,A_3}: available power in area 3

f_{A_1,A_2}: interconnection capacity between areas 1 and 2

f_{A_1,A_3}: interconnection capacity between areas 1 and 3

f_{A_2,A_3}: interconnection capacity between areas 2 and 3

f_{A_2,A_1}: interconnection capacity between areas 2 and 1

f_{A_3,A_1}: interconnection capacity between areas 3 and 1

f_{A_3,A_2}: interconnection capacity between areas 3 and 2

$f_{A_1,T}$: supplied demand in area 1

$f_{A_2,T}$: supplied demand in area 2

$f_{A_3,T}$: supplied demand in area 3

Considering the constant values at the RHS and the variables at the LHS. The feasibility cuts can be rewritten as:

Cut 1: $f_{S_o,A_1} \geq f_{A_1,T} - f_{A_2,A_1} - f_{A_3,A_1}$

Cut 2: $f_{S_o,A_2} \geq f_{A_2,T} - f_{A_1,A_2} - f_{A_3,A_2}$

Cut 3: $f_{S_o,A_3} \geq f_{A_3,T} - f_{A_1,A_3} - f_{A_2,A_3}$

Cut 4: $f_{S_o,A_1} + f_{S_o,A_2} \geq f_{A_1,T} + f_{A_2,T} - f_{A_3,A_1} - f_{A_3,A_2}$

Cut 5: $f_{S_o,A_1} + f_{S_o,A_3} \geq f_{A_1,T} + f_{A_3,T} - f_{A_2,A_1} - f_{A_2,A_3}$

Cut 6: $f_{S_o,A_2} + f_{S_o,A_3} \geq f_{A_2,T} + f_{A_3,T} - f_{A_1,A_2} - f_{A_1,A_3}$

Cut 7: $f_{S_o,A_1} + f_{S_o,A_2} + f_{S_o,A_3} \geq f_{A_1,T} + f_{A_2,T} + f_{A_3,T}$

In order to determine the upper bound for the LOLP estimator value using the Hunter inequality, the first step is to determine the probability of violating each feasibility cut individually. The probability of violating the three first cuts corresponds to the integral of the probability distribution of the available power of areas 1, 2 and 3 to the left of the corresponding RHS. The probability of violating the fourth, fifth, sixth and seventh feasibility cuts corresponds to the integral of the convoluted probability distribution of areas 1 and 2, 1 and 3, 2 and 3, 1and 2 and 3 to the left of the corresponding RHS.

The second step consists in determining the probability that two cuts are at the same time violated.

The probability of the first two cuts being violated at the same time corresponds to the product of the probability of cut 1 and cut 2 to be individually violated since the distribution of the available power of areas 1 and 2 are independent variables. However, the probability of cuts 5 and 7 being violated is a little more complicated to obtain because the available power capacity in area 3 is in both cuts. Thereby, it would be necessary to determine the joint distribution of the variables in the cuts and use the concept of conditioned probability to determine the probability of these two cuts being violated at the same time.

However, it is possible to make this calculation easier writing these cuts as a univariate distribution. This process is illustrated for the probability of violating cuts 5 and 7 at the same time.

The variables f_{S_o,A_1} and f_{S_o,A_3} are in both feasibility cuts. Writing y as the convolution of f_{S_o,A_1} and f_{S_o,A_3} ($y = f_{S_o,A_1} * f_{S_o,A_3}$) it's possible to rewrite these cuts as:

Cut 5: $y \geq f_{A_1,T} + f_{A_3,T} - f_{A_2,A_1} - f_{A_2,A_3}$

Cut 7: $y + f_{S_o,A_2} \geq f_{A_1,T} + f_{A_2,T} + f_{A_3,T}$

They can be also rewritten as:

Cut 5: $y \geq k_1$

Cut 7: $y \geq k_2 - f_{S_o,A_2}$

Where $k_1 = f_{A_1,T} + f_{A_3,T} - f_{A_2,A_1} - f_{A_2,A_3}$ and $k_2 = f_{A_1,T} + f_{A_2,T} + f_{A_3,T}$

Writing the feasibility cuts as infeasibility cuts:

Cut 5: $y \leq k_1$

Cut 7: $y \leq k_2 - f_{S_o,A_2}$

The minimum value between the integral of these two infeasibility cuts corresponds to the probability of these two cuts being violated at the same time.

5.5.3.2 Accessing lower bounds for the LOLP estimator

Once an upper bound for the LOLP estimator of each strata is determined by the Hunter inequality, a lower bound for the strata LOLP estimator value should be defined.

In the previous section (5.5.3.1) it was indicated that a lower bound could be:

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \mathcal{S}_1 - \mathcal{S}_2$$

Where:

$\mathcal{S}_1 = \sum_{i=1}^{n} P(A_i)$ and $\mathcal{S}_2 = \sum_{1\leq i\leq j\leq n} P(A_i \cap A_j)$

However, this is a weak bound [44] but Dawson and Sankoff [45] prove that the best lower limit (l) is given by:

$$l = 1 + \left\lfloor \frac{2\mathcal{S}_2}{\mathcal{S}_1} \right\rfloor \tag{48}$$

Where $\lfloor\ \rfloor$ indicates rounding down of the ratio.

5.5.3.3 Limits for LOLP estimator value and computational effort

The previous sections introduced the method for estimating lower (LB) and upper (UB) bounds for the LOLP estimator. Determining these limits is useful since they can be used as an estimation for the LOLP estimator value and then, can be used to allocate Monte Carlo draws among strata.

In addition to allow defining the quantity of draws to be allocated among strata, determining a lower bound and upper bound for the LOLP estimator of a stratum can be used to avoid the application of IS and MCMC to all strata.

The first step after determining the LB and UB of a stratum is to compare them. If they are too close there's no need to apply MCMC and IS to this stratum since the LOLP value is already known. The expected LOLP value of this stratum can be approximated by the average value between its LB and UB.

Then, a second step can be made to check if there is still any stratum in which is not "necessary" to apply MCMC and IS. If the upper limit value of the LOLP in a stratum is much lower than the system LOLP upper bound, it means that the LOLP of this stratum does not significantly contribute to the LOLP of the system and therefore, it would not be necessary to apply IS and MCMC to this stratum. So, the upper bound of each stratum, (considering its probabilities) should be compared to the LOLP UB of the system to check if each stratum could be eliminated, without the need to apply IS and MCMC to this stratum.

Hence, to eliminate strata, i.e., neither considering for the IS nor MCMC, result in a reduction of the computational effort since eliminating strata reduces the simulation time required for the reliability analysis of the system.

Then, it's possible to conclude that defining bounds for the LOLP value has two main advantages:

- Defining the number of Monte Carlo draws to be allocated among the strata according to the LOLP estimated value in each stratum and,
- Possibility to "disregard some strata", i.e., do not apply MCMC and IS to these strata that do not contribute into the system load-shedding scenarios.

5.5.4 Demand Representation with Renewable Treatment

Traditionally in reliability studies it was assumed that most load loss events occurred in periods of heavy load, because the "margin of reserve" in the system is smaller. However, with the increase of intermittent renewable sources in the systems' energy matrix, it's even more likely that supply failure occurs at times of medium or light load, depending on the contribution of renewable sources in the system.

Moreover, for reliability studies it's important to consider a temporal dependence between renewable generation and demand, and a possible spatial correlation between renewables and demand.

Once the stratification is applied for the renewable generation, bins are defined for the demand curve in each stratum. It's important to highlight that the definition of demand bins inside each stratum of each area allows to maintain any temporal or spatial correlation between demand and renewable generation inside one area.

5.5.5 Monte Carlo Simulation with Strata

In this work, stratification is the proposed method to treat intermittent sources in reliability studies. The proposed method described in the previous chapters (MCMC and IS Optimization in bins) is then applied for each stratum individually.

In other words, samples of MCMC will be determined for each stratum and allocated to the bins of generation capacity and bins of demand in each area. Thereby, an importance distribution is obtained for each bin in each stratum and then, for a given stratum, a sample of Monte Carlo simulation is produced by the following steps:

1. Sample a bin of total aggregated demand and a bin of the aggregated generating capacity of each area, considering the importance distributions determined through the IS optimization.
2. Calculate demand LR for the demand bin; from the hours within the bin, draw one hour and set corresponding area demands.
3. For each area, sample a generating capacity from its original distribution, truncated outside the sampled bin and calculate the LR.

4. Calculate the joint LR for the sampled state, that is the demands and generating capacities of the areas.
5. Evaluate que loss of load for the sampled state; update LOLP and other estimators and the corresponding uncertainties, according to Equation (24).

Once the LOLP of each stratum is determined, the system LOLP is calculated considering the sum of the LOLP value in each stratum conditioned to its probability.

6 Results

In this chapter some results obtained applying the proposed methodology in two case studies are presented. These two cases derive from real systems. The first one is based on the Saudi Arabia system and does not include renewables and then, applies the concepts of IS and MCMC. The second case study applies IS, MCMC and Stratification methodology to a Chile-derived system with great insertion of renewable generation sources.

For the sake of highlighting the performance and the effectiveness of the proposed methodology, in both case studies, the LOLP value obtained through the proposed methods is compared to the value determined with the Standard Monte Carlo Simulation.

6.1 Case Study – Saudi Arabia

In this section the concepts of MCMC and IS optimization introduced in this work are applied in a system based in the Saudi Arabia system which was divided into four electric areas, each one with an hourly demand profile proportional to the total demand of the system, as can be observed in Figure 27.

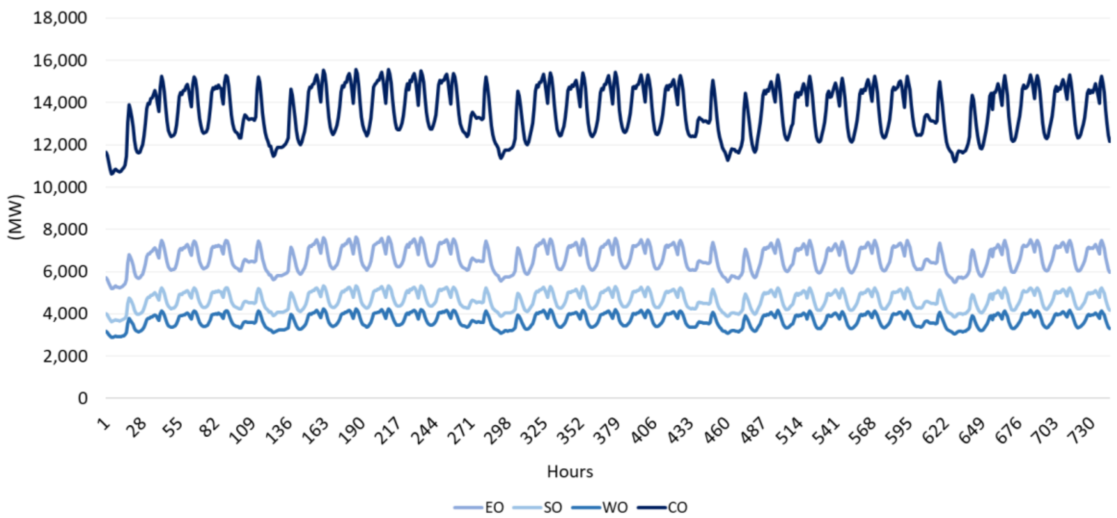


Figure 27: One-month demand profile in each area

These four electric areas are interconnected by four transmission lines. The total available power in each area and the interconnection capacity among them are indicated in Table 18 and Table 19.

Table 18: Total Available Power in each of the four areas

Areas	Total Available Power (MW)
Eastern (EO)	18,520
Southern (SO)	4,194
Western (WO)	15,055
Central (CO)	9,530

Table 19: Interconnection Capacities among the four areas

Areas	Interconnection Capacity (MW)
Eastern - Central	4,500
Central - Western	2,000
Southern - Western	2,000
Southern - Central	2,000

In this system there are thermal and hydro plants as sources of energy. However, hydro sources are represented in a simpler way in the reliability assessment since their availability does not consider hydrology or their reservoir volume throughout the year.

Using the k-means algorithm, the total demand of Saudi Arabia was divided into 20 clusters or bins and the total available power of EO, SO, WO and CO in 15, 9, 14 and 11 bins respectively.

Then, a set of 5,000 infeasible samples (load shedding scenarios) were obtained through MCMC to obtain infeasible samples. Table 20 and Table 21 indicate the frequency of MCMC samples allocated to each bin in each area.

Table 20: Number of samples in bins of each area (obtained through MCMC)

Area/ Bin	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
EO	-	6	14	26	35	45	51	81	98	121	111	135	113	96	68
SO	457	383	119	29	6	4	1	1	-						
WO	1	5	9	19	26	45	70	77	99	125	92	115	127	190	
CO	40	212	232	185	144	87	52	34	11	2	1				

Table 21: Number of samples in demand bins (obtained through MCMC)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Load	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	37	962

From the relative frequency of the samples in each bin, it's determined the importance probability of each bin in each area. The graphs from Figure 28 to Figure 32 indicate the original and importance probability distributions for each area and for the total demand of the system. In addition, the range of values inside each bin is indicated for each area.

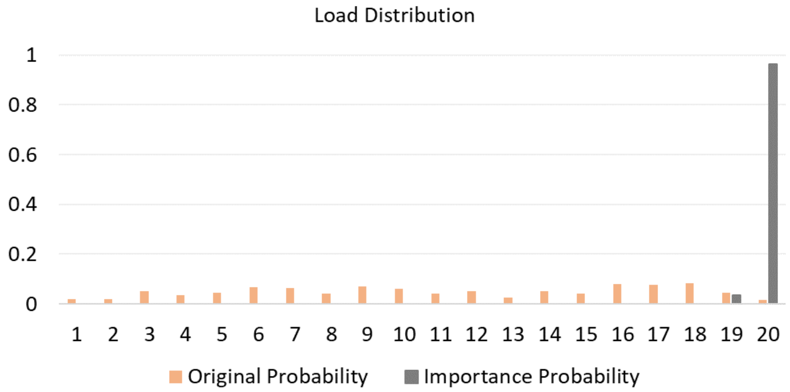


Figure 28: Original and Importance Distribution of the system load

Load Bin	Total Load Range
1	[0, 22670]
2	(22670, 23975]
3	(23975, 24664]
4	(24664, 25182]
5	(25182, 25725]
6	(25725, 26182]
7	(26182, 26636]
8	(26636, 27059]
9	(27059, 27572]
10	(27572, 28054]
11	(28054, 28698]
12	(28698, 29202]
13	(29202, 29653]
14	(29653, 30135]
15	(30135, 30487]
16	(30487, 30798]
17	(30798, 31176]
18	(31176, 31547]
19	(31547, 32096]
20	(32096, 32765]

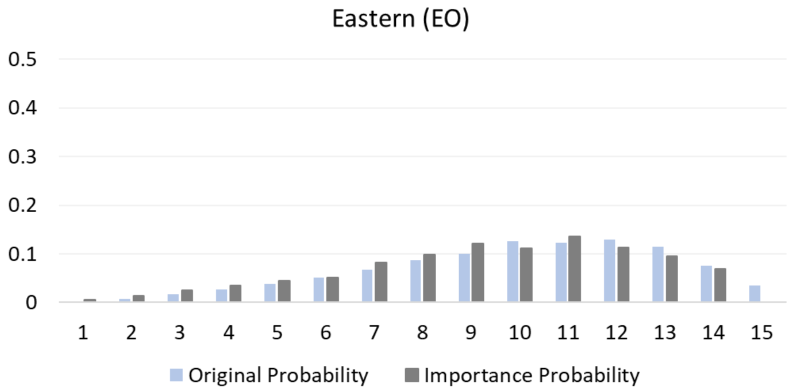


Figure 29: Original and Importance Distribution of the Eastern Area

EO	Total Available Power
1	(0,15129]
2	(15129,15843]
3	(15843,16173]
4	(16173,16430]
5	(16430,16627]
6	(16627,16797]
7	(16797,16949]
8	(16949,17099]
9	(17099,17241]
10	(17241,17392]
11	(17392,17530]
12	(17530,17668]
13	(17668,17813]
14	(17813,17974]
15	(17974,18520]

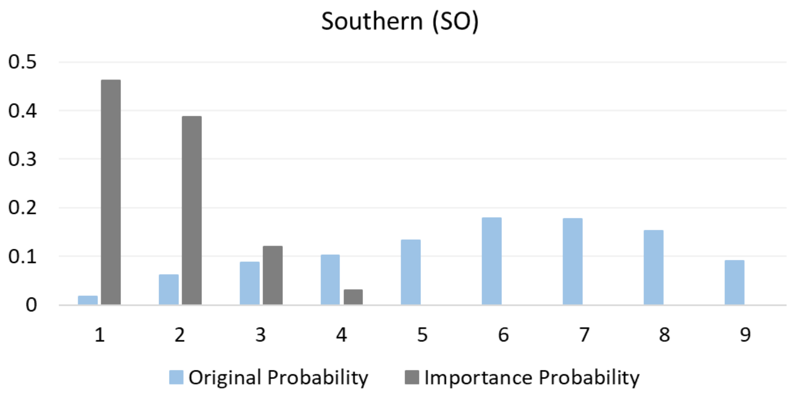


Figure 30: Original and Importance Distribution of the Southern Area

SO	Total Available Power
1	(0,3351]
2	(3351,3579]
3	(3579,3702]
4	(3702,3803]
5	(3803,3883]
6	(3883,3949]
7	(3949,4004]
8	(4004,4058]
9	(4058,4194]

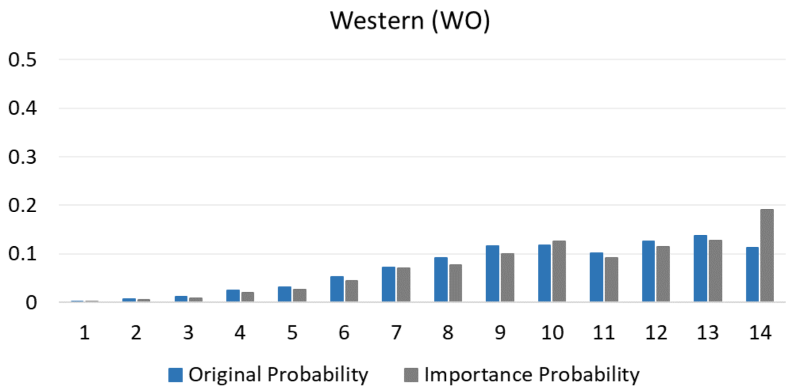


Figure 31: Original and Importance Distribution of the Western Area

WO	Total Available Power
1	(0,12966]
2	(12966,13473]
3	(13473,13683]
4	(13683,13867]
5	(13867,14017]
6	(14017,14151]
7	(14151,14279]
8	(14279,14405]
9	(14405,14510]
10	(14510,14613]
11	(14613,14715]
12	(14715,14806]
13	(14806,14885]
14	(14885,15055]

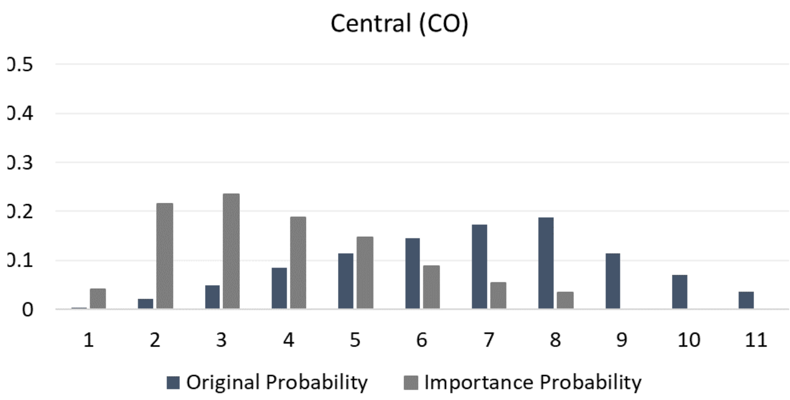


Figure 32: Original and Importance Distribution of the Central Area

CO	Total Available Power
1	(0,8496]
2	(8496,8688]
3	(8688,8798]
4	(8798,8882]
5	(8882,8952]
6	(8952,9015]
7	(9015,9080]
8	(9080,9149]
9	(9149,9216]
10	(9216,9282]
11	(9282,9530]

Once the importance probability of each bin in each area is determined with the Importance Sampling Optimization considering the load shedding scenarios obtained by

MCMC, Monte Carlo method is applied to sample bins. Given a sampled bin, a generation capacity state from the corresponding truncated distribution of that bin is sampled and thereby, it's calculated the supply adequacy for the sampled state. For each Monte Carlo sample the reliability indexes are updated. This process is repeated until the convergence is reached.

In this example, there were considered 10,000 Monte Carlo draws and the number of draws was used as the convergence criterion. In other words, with 10,000 Monte Carlo draws the obtained LOLP estimator value is 0.0001 with a relative uncertainty of 4.4%. Figure 33 indicates the LOLP estimator convergence process over the first 1,000 samples of the Monte Carlo Simulation.

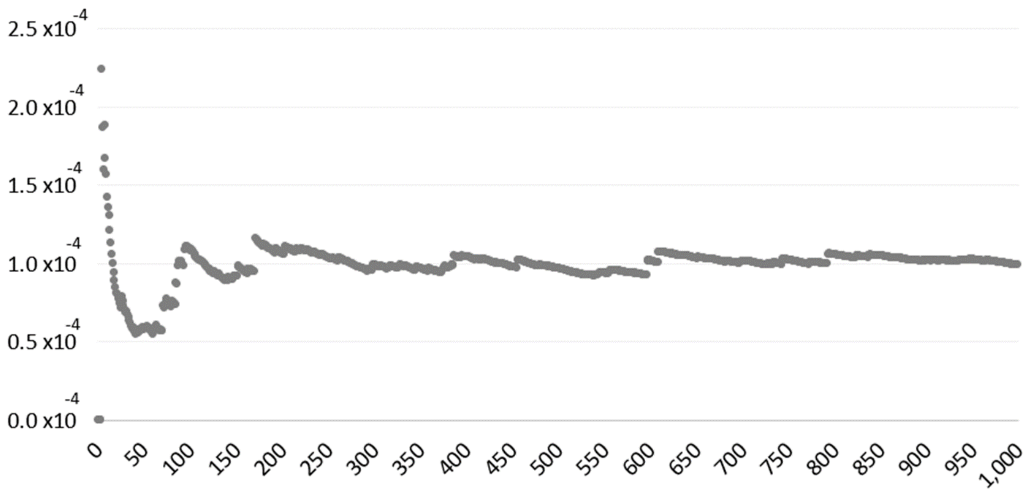


Figure 33: LOLP estimator convergence – Saudi Arabia based system

For the sake of comparing the computational time and the expected value of LOLP obtained with the proposed method, the same system was simulated by the traditional Monte Carlo method, until the same coefficient of variation of 4.4% was obtained. 5,000,000 draws were necessary to achieve an accuracy of 4.4% and the resulting LOLP value was 0.0001.

Table 22 summarizes the main results of the simulations using the traditional Monte Carlo Method and the method proposed in this work (MCMC + IS + Monte Carlo).

Table 22: Comparison of proposed method and Standard Monte Carlo Method

	LOLP expected value	Relative Uncertainty	Number of samples
Standard MC	0.0001	4.4%	5,000,000
MCMC+IS+MC	0.0001	4.4%	15,000[7]

[7] It corresponds to the number of MCMC and MCS samples.

Therefore, the proposed method let obtaining the same LOLP estimator value as determined by the Standard Monte Carlo Method, but with a speed-up of more than 300 times. It highlights the efficiency of the proposed method.

6.2 Case Study – Chile

In this section, the concepts presented in this thesis are applied in a system based on the Chilean system. This system is divided in three electrical areas, interconnected by 2 circuits whose power capacities are 2500 MW and 1200 MW as illustrated in Figure 34.

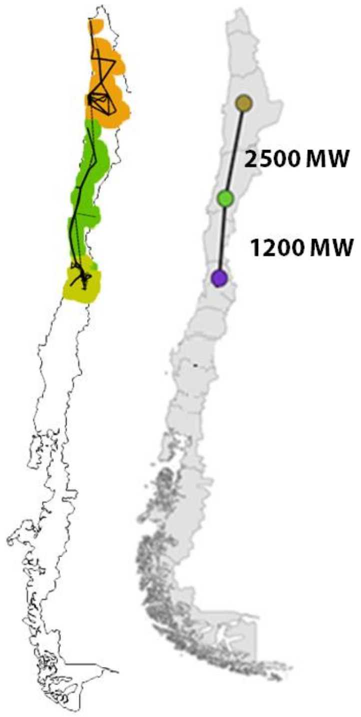


Figure 34: 3 Multi-Area considered in the case study based on the Chilean system

The number of generation units in each area of the three Multi Area system and their total installed capacity are indicated in Table 23 and Table 24.

Table 23: Number of generation units in the system

Area	Thermal	Hydro	Renewables
1	41	23	13
2	45	-	17
3	21	4	40

Table 24: Total installed capacity of generators in the system (MW)

Area	Thermal	Hydro	Renewables

1	3,643	1,149	195
2	4,708	-	832
3	1,176	33	1,653

The Chilean energy matrix is made up of thermal, hydroelectric and renewable generation sources. Hydro plants are mostly run-of-the-river so available capacity disregards the effect of depletion of reservoir volume. For this system the hourly demand in each area does not follow the total demand load curve, therefore the probability distribution of the total demand is only used to derive the importance distribution.

For the reliability study, 44 renewable generation scenarios were used. In this section the reliability indexes were calculated for January, although this analysis can be replicated for any other months of the year.

6.2.1 Stratification of Renewables

The system considered in the simulation has 44 hourly generation profiles for each day of the year, for each renewable plant. In other words, for the month of January, there are for each hour 1374 renewable generation values for each plant (44 scenarios x 31 days).

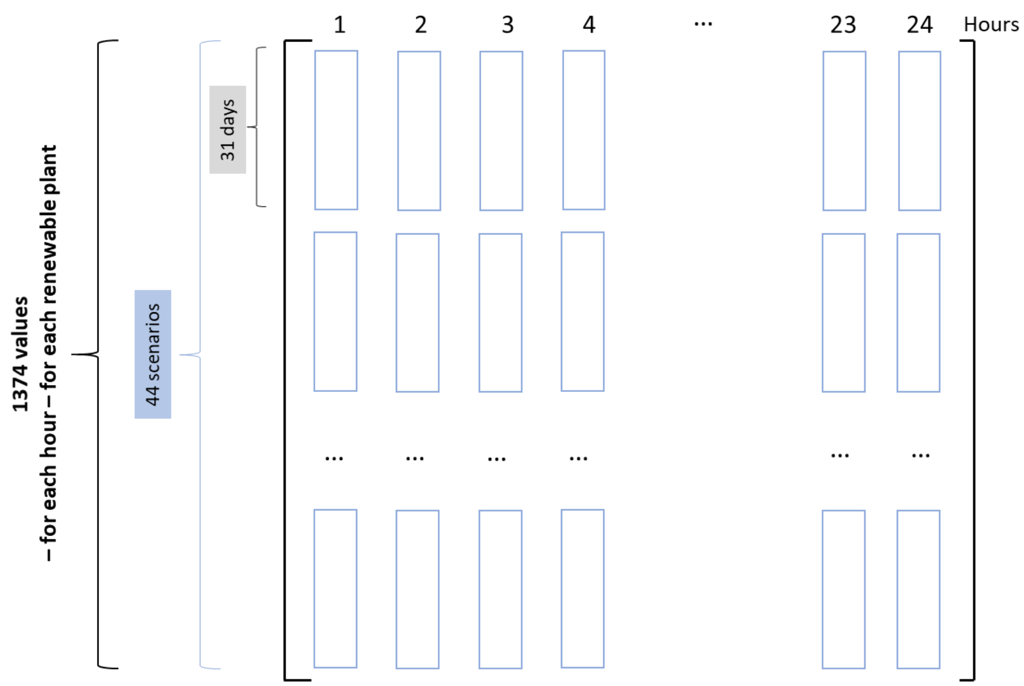


Figure 35: Scheme for available data for hourly generation profiles

For the sake of applying the stratification method to these scenarios and define strata in which the available power of renewables sources would be stationary, the generation profile of these renewable sources should be aggregated and analyzed.

Then, for each one of these 44 scenarios, the hourly profile of all renewable plants was added, resulting in a scalar value for each one of the 24 hours of each scenario.

Then, the mean and CVaR (*Conditional Value at Risk*) values were calculated for each hour of these 44 scenarios. The renewable generation value for each hour of the daily cycle was then defined by the convex combination of the expected value and the CVaR (70% for the expected value and 30% for the CVaR), i.e.:

$$G_h = \lambda\, E_h + (1 - \lambda) CVaR_h \qquad (49)$$

Where:

G_h is the total renewable generation at hour h

λ is the parameter that multiplies the expected value (λ = 70%)

E_h is the expected value for the 44 scenarios of hour h

$CVaR_h$ is the $CVaR$ of the 44 scenarios for hour h

Table 25 illustrates the values of renewable generation for each hour, obtained from the convex combination.

Table 25: Renewable generation profile – Convex Combination

Hour	Average	CVaR (95%)	Convex Combination
1	536	548	540
2	471	482	474
3	409	418	412
4	392	400	394
5	368	375	370
6	345	352	347
7	330	337	332
8	989	1,010	995
9	1,200	1,220	1,206
10	1,410	1,430	1,416
11	1,480	1,490	1,483
12	1,510	1,520	1,513
13	1,490	1,500	1,493
14	1,480	1,480	1,480
15	1,500	1,510	1,503
16	1,520	1,540	1,526
17	1,590	1,610	1,596
18	1,610	1,630	1,616
19	1,490	1,510	1,496
20	1,100	1,120	1,106
21	651	664	655
22	693	707	697
23	693	707	697
24	670	685	675

From the convex combination values, strata are defined based on "homogeneous" profiles in consecutive hours. For this system, four strata were defined according to Table 26, based on the profile shown in Figure 36:

Table 26: Hours within each stratum - Chilean System

Renewable Strata	Hours of the day
1	8 A.M – 9 A.M
2	9 A.M – 7 P.M
3	8 P.M – 9 P.M
4	9 P.M – 7 A.M

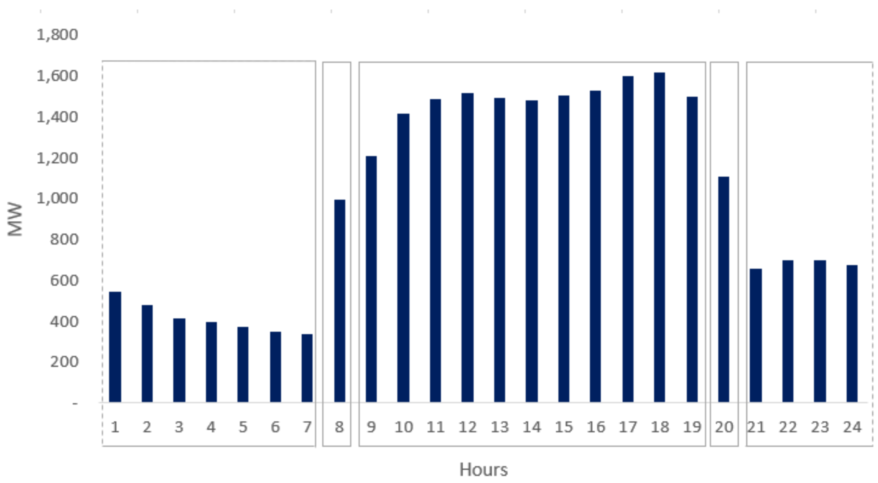


Figure 36: Definition of strata based on the generation profiles of renewable sources

So, for each of the three areas, there are four strata for the renewable generation which will be analyzed individually.

6.2.2 Calculating Bounds for Each Strata

The stratification method allows representing the variability of renewable energy sources and lets obtaining groups with stationary distribution of their available power. Since, each of these strata has stationary distribution the total available power for each stratum can be obtained by numerical convolution of the renewable power distribution of each stratum and the available thermal and hydro distribution of each area[8].

The numerical convolution results in 4 total available power distribution in each area, each one, corresponding to the distribution of each stratum.

[8] As mentioned, in this work hydro plants available capacity disregards the reservoirs volume and inflows. Thereby, hydro plants are treated similarly as thermal plants. In the future work section is proposed a methodology to represent hydro plants in a more accurate way.

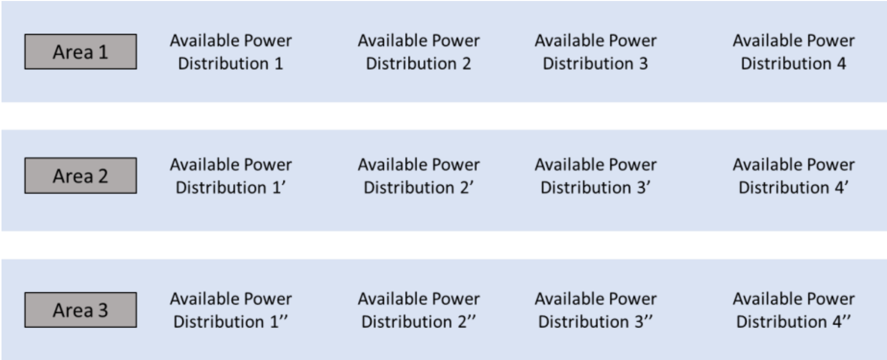


Figure 37: Scheme with the number of available power distributions in each area

Once the total available power distribution is obtained for each stratum, MCMC and IS technique could be applied. However, as explained in previous sections, one can avoid applying the MCMC and IS methods to strata that do not contribute to the system reliability indexes.

Then, once obtaining the total available power of each strata, it's possible to estimate upper and lower limits for the LOLP estimator and evaluate if some stratum can be disregarded from the sampling process of MCMC and IS.

6.2.2.1 Hunter inequality for the Chilean system

For the sake of determining an upper bound estimation for the LOLP of each stratum, the Hunter inequality is applied for each stratum. This upper bound can be used to disregard some stratum from the reliability analysis if its LOLP value doesn't contribute for the system's probability of failure.

Since the Chilean system has 3 electrical areas, there are seven possible failure modes $(2^3 - 1)$. For each stratum, the failure probability of each failure mode varies because the renewable generation and demand varies among strata. Therefore, for each stratum it is necessary to apply the Hunter inequality.

- **<u>Analysis of the stratum</u>**

In this section the values obtained for the Hunter inequality of each stratum are indicated. The whole process is exemplified for the stratum 1 because it has the smallest number of hours (as well as stratum 3).

Stratum 1

The “Stratum 1” comprises hours from 7 A.M until 8 A.M. Then this stratum has 31 demand observations (31 days x 1 hours). For each of these observations, the Hunter inequality is applied, i.e., for each value of demand, fixing the interconnection capacities, an upper limit and lower limit for the LOLP value is determined.

Table 27: LOLP UB and LB for each hour in stratum 1

	Hour	Total Load	Load Area 1	Load Area 2	Load Area 3	Lower Bound	Upper Bound
1	632	6070.6	3240.5	2146.5	683.6	8.10E-07	8.68E-07
2	128	6213.1	3361.6	2142.3	709.2	5.79E-05	5.79E-05
3	464	6247.5	3370.0	2166.6	710.9	3.01E-04	3.01E-04
4	296	6348.0	3353.8	2286.7	707.5	1.11E-03	1.11E-03
5	608	6983.4	4006.1	2132.1	845.1	3.54E-06	3.54E-06
6	8	7124.3	4020.3	2255.9	848.1	6.24E-08	8.22E-08
7	104	7181.8	4073.7	2248.7	859.4	2.09E-04	2.09E-04
8	272	7218.2	4066.6	2293.7	857.9	4.73E-04	4.73E-04
9	440	7257.0	4128.4	2257.6	870.9	1.56E-04	1.56E-04
10	32	7358.0	4284.3	2169.9	903.8	1.88E-04	1.88E-04
11	728	7367.8	4371.7	2073.9	922.2	5.84E-04	5.84E-04
12	392	7406.6	4528.3	1923.1	955.3	3.12E-06	3.12E-06
13	656	7472.4	4354.3	2199.5	918.6	6.24E-08	8.22E-08
14	200	7492.0	4392.1	2173.4	926.5	1.88E-04	1.88E-04
15	224	7496.6	4413.5	2152.1	931.1	4.34E-04	4.34E-04
16	560	7517.9	4448.1	2131.5	938.3	5.39E-04	5.39E-04
17	56	7520.3	4465.6	2112.7	942.0	5.12E-04	5.12E-04
18	512	7585.1	4466.6	2176.2	942.3	3.94E-04	3.94E-04
19	584	7586.6	4436.7	2214.0	936.0	9.19E-06	9.19E-06
20	344	7610.6	4509.9	2149.3	951.4	6.24E-08	8.22E-08
21	368	7614.9	4532.1	2126.7	956.1	2.72E-04	2.72E-04
22	488	7615.1	4453.4	2222.2	939.5	3.01E-04	3.01E-04
23	704	7631.3	4471.4	2216.6	943.3	4.73E-04	4.73E-04
24	680	7653.3	4429.7	2289.2	934.5	2.63E-04	2.63E-04
25	176	7662.5	4520.0	2188.9	953.5	2.35E-04	2.35E-04
26	320	7666.4	4416.9	2317.7	931.8	5.32E-07	5.32E-07
27	416	7686.2	4498.6	2238.5	949.0	6.24E-08	8.22E-08
28	152	7692.0	4425.0	2333.4	933.5	1.11E-04	1.11E-04
29	536	7706.6	4518.6	2234.7	953.2	2.22E-04	2.22E-04
30	80	7771.4	4627.4	2167.9	976.2	3.15E-04	3.15E-04
31	248	7815.7	4545.9	2310.8	959.0	1.28E-04	1.28E-04

The estimation for the LOLP estimator upper bound (UB) and lower bound (LB) corresponds to the average values of the upper and lower bounds obtained for the 31 observations.

Table 28: Average LOLP limits in stratum 1

Hourly Average LOLP UB = **2.42 E-04**
Hourly Average LOLP LB = **2.41 E-04**
Stratum probability = 1/24 = **0.0147**

Stratum 2

The "Stratum 2" comprises hours from 9 A.M until 7 P.M. Then, this stratum has 341 demand observations (31 days x 11 hours). For each of these observations, the Hunter inequality is applied, fixing the interconnection capacities.

The estimation for the LOLP estimator upper bound (UB) and lower bound (LB) corresponds to the average values of the upper and lower bounds obtained for the 341 observations.

Table 29: Average LOLP limits in stratum 2

Hourly Average LOLP UB = **6.01 E-04**
Hourly Average LOLP LB = **6.00 E-04**
Stratum probability = 11/24 = **0.458**

Stratum 3

The "Stratum 3" comprises hours from 7 P.M until 8 P.M. Then, this stratum has 31 demand observations (31 days x 1 hours). For each of these observations, the Hunter inequality is applied, fixing the interconnection capacities. Table 30 indicates the estimation for the LOLP estimator upper bound (UB) and lower bound (LB).

Table 30: Average LOLP limits in stratum 3

Hourly Average LOLP UB = **1.54 E-3**
Hourly Average LOLP LB = **5.40 E-4**
Stratum probability = 1/24 = **0.0457**

Stratum 4

The "Stratum 4" comprises hours from 9 P.M until 7 A.M. Then, this stratum has 341 demand observations (31 days x 11 hours). For each of these observations, the Hunter inequality is applied, fixing the interconnection capacities. Table 31 indicates the estimation for the LOLP estimator upper bound (UB) and lower bound (LB).

Table 31: Average LOLP limits in stratum 4

Hourly Average LOLP UB = **1.72 E-05**
Hourly Average LOLP LB = **1.71 E-05**
Stratum probability = 11/24 = **0.458**

The hourly average LB and UB for each stratum can be summarized in Table 32. It's possible to observe that LB and UB for stratum 1,2 and 4 are very close, and then, the expected value for LOLP of these strata could be approximate to the average of the LB and UB for each stratum. Therefore, there's no need to apply MCMC and IS for these three strata.

Table 32: LB and UB comparison

Stratum	Lower Bound (LB)	Upper Bound (UB)	UB/LB (%)
1	2.41E-04	2.42E-04	**0.4%**
2	6.00E-04	6.01E-04	**0.2%**
3	5.40E-04	1.54E-03	**185%**
4	1.71E-05	1.72E-05	**0.6%**

The second step correspond to analyze if there is still any stratum that would not be necessary to apply MCMC and IS. Table 33 indicates the lower and upper bound of each stratum and the probability of each one.

Table 33: UB x duration of the remaining stratum vs. Total UB

Stratum	Lower Bound (LB)	Upper Bound (UB)	Probability (Stratum duration)	UB x Duration
1	2.41E-04	2.42E-04	0.042	1.01E-05
2	6.00E-04	6.01E-04	0.458	2.75E-04
3	5.40E-04	1.54E-03	0.042	6.42E-05
4	1.71E-05	1.72E-05	0.458	7.88E-06
			Total UB:	**3.58E-04**

Since the product of the LOLP upper bound and the stratum probability of the "remaining stratum" can't be neglected (stratum 3), since this product value is not much smaller than the system's lower bound, MCMC must be applied in this stratum to obtain load shedding scenarios that will be used in the IS optimization of the bins in this stratum.

6.2.3 Demand Representation

Since LOLP estimator limits (UB and LB) were close for all strata except to stratum 3, MCMC and IS just to stratum 3. Thereby, k-means algorithm is just applied to stratum 3 which result in 4 bins for the demand. It is important to note that the probability of the bins of demand should not be small, since small probabilities reduce the chances of sampling events in that bin during the MCMC.

6.2.4 Monte Carlo Markov Chain

Once calculating the upper bounds (applying the Hunter inequality) and lower bounds for each stratum, it's defined the strata to be analyzed and then, MCMC is applied to each stratum to obtain scenarios of loss of load from a given failure sample. MCMC is applied in the joint distribution of all areas and the available power of each area in the failure scenarios is associated to the bins of available power and demand of each area.

The demand for stratum 3 was divided in 4 bins as previous mentioned.

The total available power of each area in stratum 3 was divided in bins according to Table 34.

Table 34: Number of bins of power capacity in each area – Stratum 3

	Number of bins of available power capacity
Area 1: CE (A_1)	10
Area 2: NG (A_2)	10
Area 3: NO (A_3)	5

In this thesis there were considered 5,000 MCMC scenarios for the stratum. The MCMC samples are shown for stratum 3.

<u>Stratum 3</u>

The 5,000 events sampled with the MCMC are allocated in bins of the stratum 3 according to the bins range.

Table 35 and Table 36 indicate, respectively, the number of MCMC samples in each bin of demand and bin of capacity in each area.

Table 35: MCMC samples allocated to bins of demand – Stratum 3

	Number of samples
D_1	4
D_2	29
D_3	268
D_4	949

Table 36: MCMC samples allocated to power capacity bins – Stratum 3

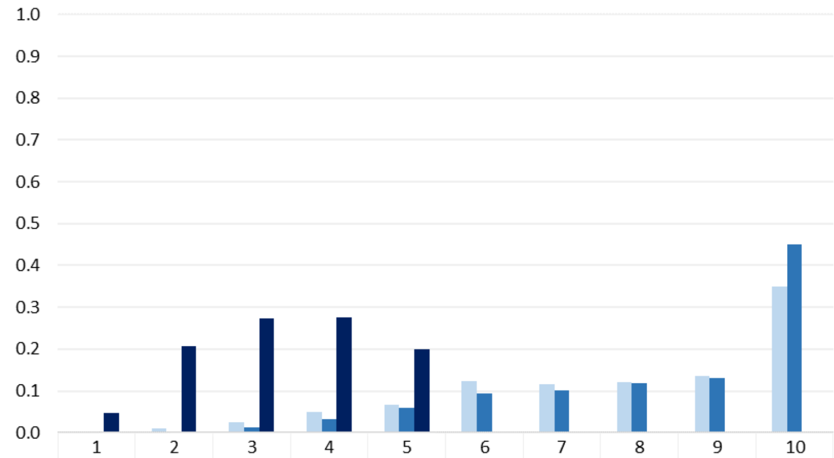
	Bin 1	Bin 2	Bin 3	Bin 4	Bin 5	Bin 6	Bin 7	Bin 8	Bin 9	Bin 10
A_1	857	392	-	-	-	1	-	-	-	-
A_2	1	1	2	9	15	41	57	73	69	982
A_3	62	263	338	348	239	-	-	-	-	-

6.2.5 Importance Sampling

Once scenarios that result in load shedding in the system are obtained through the MCMC in stratum 3, the relative frequency of the draws in each bin, obtained by MCMC, can be determined according to Equation (22).

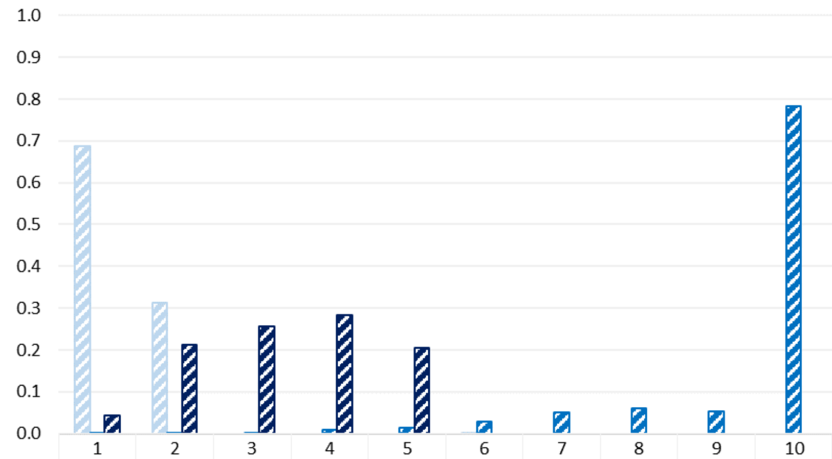
Therefore, by IS optimization importance probabilities are determined.

The graphs in Figure 38, Figure 39 and Figure 40 illustrates the original and importance probabilities for the bins in stratum 3 in which MCMC and IS were applied.



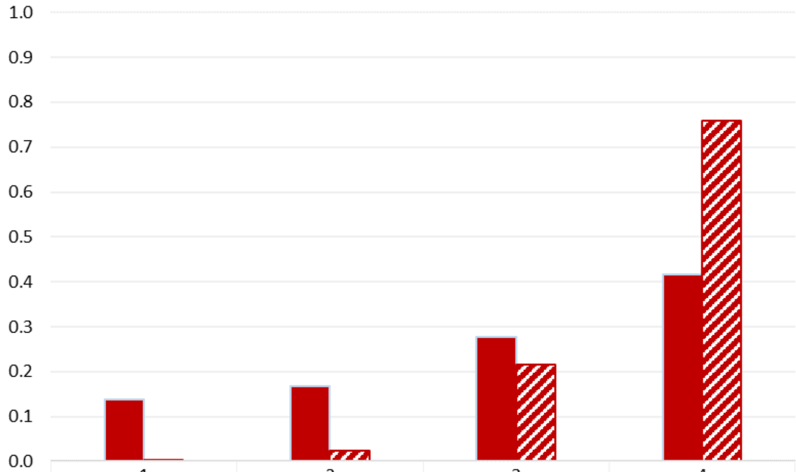
	1	2	3	4	5	6	7	8	9	10
CE (A1)	0.001	0.011	0.026	0.049	0.067	0.124	0.117	0.120	0.137	0.350
NG (A2)	0.000	0.003	0.013	0.032	0.059	0.094	0.100	0.120	0.131	0.450
NO (A3)	0.047	0.206	0.272	0.275	0.200					

Figure 38: Original probability of the bins in each area for stratum 3



	1	2	3	4	5	6	7	8	9	10
CE (A1)	0.688	0.312				0.000				
NG (A2)	0.000	0.000	0.003	0.009	0.015	0.028	0.051	0.060	0.052	0.782
NO (A3)	0.044	0.212	0.257	0.282	0.205					

Figure 39: Importance probability of the bins in each area for stratum 3



	1	2	3	4
Original	0.137	0.168	0.277	0.417
Importance	0.004	0.023	0.214	0.759

Figure 40: Original and Importance probability of demand bins for stratum 3

6.2.6 Standard Monte Carlo

The IS optimization technique allows to obtain an importance probability of each bin of generation available power and demand, in each stratum. In this example, importance probabilities were obtained for bins in stratum 3. Samples from these importance probabilities are obtained to assess the reliability adequacy of a system through the Standard Monte Carlo simulation.

Since a importance probabilities are being considered to obtain the samples for the reliability evaluation, the likelihood ratio must be considered in order to preserve the expected value for the reliability indexes of a system.

In this thesis it’s considered 50,000 samples of Monte Carlo that can be drawn from the importance probabilities. As demonstrated in section 5.2.2, the number of samples drawn in each stratum is proportional to the product of the LOLP estimator standard deviation and to the probability (duration) of each stratum. Since just stratum 3 is considered for the Monte Carlo Simulation, 5,237 samples were drawn from stratum 3.

The first step of the Monte Carlo Simulation is to sample one bin in each area for one stratum. Then, for each bin, an available power and demand values are sampled. Thus, an adequacy analysis is conducted to check if these samples result or not in load shedding in the system. The Monte Carlo process can be summarized by Figure 41.

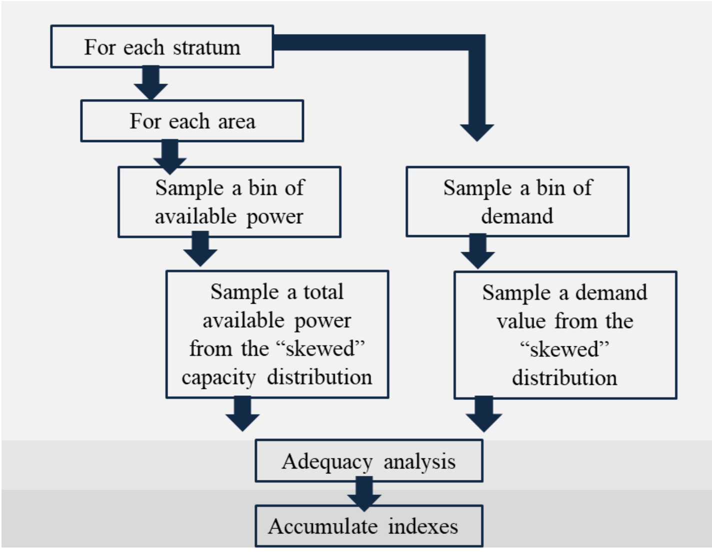


Figure 41: Monte Carlo Simulation Process

Thereby, once sampling 5,237 draws using the Standard Monte Carlo method, a LOLP expected value is obtained for stratum 3. For the sake of determining the expected LOLP value for the system, the expected LOLP of each stratum is weighted by the duration of each stratum obtaining the total LOLP of the system.

Table 37 indicates the LOLP values for each stratum and the probability of the strata, in addition to the total LOLP of the system. The value of the relative uncertainty in the calculation of the LOLP expected value (α as mentioned in section 3.2) can be calculated as the ratio between its standard deviation and average values.

Table 37: LOLP value considering the proposed methodology

Stratum	Probability (Stratum duration)	LOLP value	Product
1	0.042	2.42E-04	1.01E-05
2	0.458	6.01E-04	2.75E-04
3	0.042	1.52E-03	6.33E-05
4	0.458	1.72E-05	7.87E-06
System expected LOLP:			**3.56 E-04**
Relative uncertainty			**1.8%**

The LOLP expected value of stratum 1, 2 and 4 were determined considering the average values of each LB and UB, while the LOLP expected value for stratum 3 was determined considering MCMC, IS and the Standard Monte Carlo.

The graph in Figure 42 illustrates the LOLP convergence for Stratum 3 during the first 5,000 samples of the Standard Monte Carlo Simulation. From it can be observed the fast convergence to the LOLP expected value for this stratum.

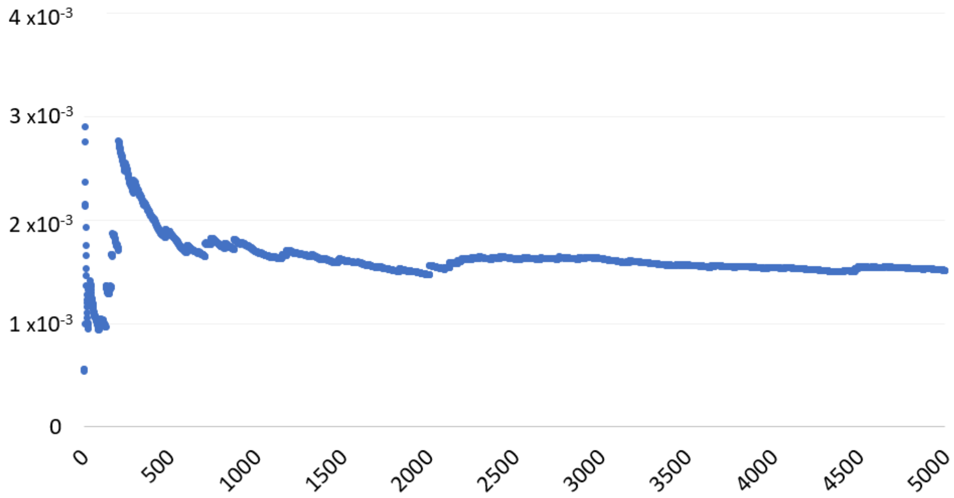


Figure 42: LOLP estimator convergence of stratum 3– Chilean based system

6.2.7 Speed-up Analysis

The methodology proposed in this thesis to access the reliability of a multi-area system includes applying stratification to represent renewable sources and MCMC and IS optimization techniques.

For the sake of comparing this methodology with the Standard Monte Carlo Simulation method, a speed-up analysis is performed to compare the efficiency of these two methods.

Once the LOLP expected value and the relative uncertainty are known (1.8%), it's possible to determine that approximately 8.9 million draws (Equation (3)) would be required to determine the same LOLP expected value, with this same relative uncertainty, considering the Standard Monte Carlo Method.

The speed up can be determined by comparing the number of draws that would be required by the Standard Monte Carlo Method with the number of draws used with the proposed methodology.

Table 38: Speed-up analysis – Case Study of the Chilean System

Stratum	LOLP expected value	Number of samples
Standard MC Simulation	3.56 E-04	8,700,000
Proposed Method: MCMC+IS+MC	3.56 E-04	10,237

This comparison results in a speed up of more than 850 times.

Then, this reduction in computational time evidences the efficiency of the method proposed in this work.

7 Conclusions

This work proposes a new methodology for power systems reliability assessment in multi-area systems. The standard Monte Carlo simulation is a well-known method for power system reliability assessment because it allows considering stochastic scenarios in the electrical systems and the number of samples required to calculate the reliability indexes do not depend on the complexity of the system. However, the number of samples is invert proportional to the indexes expected value and it is direct proportional to its variance value. Since electrical systems are usually very reliable, it may take too many draws to sample load shedding scenarios that will contribute to accumulate the indexes.

Therefore, this work considered a new multi-area reliability methodology and Importance Sampling in truncated subsets of the total available power distribution (bins) of each area. For the sake of determining the optimum importance sampling is proved that the optimum distribution of each bin is the relative frequency of each bin in a set of load-shedding scenarios. Thereby, Markov Chain Monte Carlo is applied to generate these load shedding scenarios. This methodology significantly reduces the computational simulation time and number of samples required for reliability studies compared to the standard Monte Carlo simulation. These techniques were applied to a study derived from real power systems with just thermal sources and resulted in a speed-up of more than 300 times compared to the Standard Monte Carlo Method.

Moreover, in a context of fast insertion of renewables sources worldwide this work proposes considering stratification for an adequate model of renewable sources in power systems reliability assessment. It allows capturing particularities of renewable sources, such as its stochastic behavior, correlation with the demand of energy and any spatial correlation among renewable generation sources.

At first, stratification could lead someone to think that the computational time effort could increase due to the necessity to analyze each stratum individually. However, this work proposed a methodology to identify the stratum that should be considered for the Power Systems Assessment. This approach considers determining tights lower and upper bounds for the LOLP expected value. Then, if these values are close it means that LOLP expected value of each stratum is known “a priori” and then Importance Sampling and MCMC could not be applied in this stratum. A study derived from the Chilean power system, considering three areas and 44 stochastic renewable generation profiles illustrated that a speed up of more than 800 times is obtained with the method proposed when compared to the Standard Monte Carlo Simulation (considering the same coefficient of variation).

The application of the methodology in examples derived from real power systems highlight the effectiveness of the proposed method to assess a Power Systems Reliability once a meaningful speed-up is obtained considering a great modelling for renewable source.

7.1 Future works

Some analysis can be proposed as future works: a greater hydroelectric representation and composite reliability evaluation, considering control variables.

7.1.1 Hydroelectric Representation

Some power systems, such as Brazil and Colombia, have their energy matrixes mostly composed by hydro power plants. In the past, when there weren't intermittent renewable sources in the electric matrix, the reliability evaluation in these countries could be accessed considering many scenarios of hydro generation to consider the stochastic behavior of hydro generation. In Brazil where reservoirs are big and with great regulation capacity, it was considered that the water level in reservoirs didn't change during the months and then, for each hydrologic scenario the total available power of hydro plants were considered constant. In this context, the reliability was usually accessed aggregating the demand hours in blocks with light, medium and peak load levels.

However, with the fast insertion of renewable sources worldwide and their percentage increase in energy matrixes' participation, the reliability evaluation had not just to consider renewable power plants, but also consider small time steps during simulation.

Moreover, the reliability simulation models must be able to capture any spatial correlation among renewable and hydroelectric power plants.

For the sake of considering the stochastic characteristic of renewable sources, hydro plants contribution to load supply and preserving any spatial correlation among renewable and hydroelectric plants in reliability studies, it's suggested to define bins of hydrology and renewable scenarios and apply the IS and MCMC to each of them individually.

The steps for defining these clusters will be described below.

For one month, for each hydrologic scenario, there are 730 hours of renewable generation. It's the same to say that for each hydrologic scenario there are 30 profiles of daily profile (24 hours) of renewable generation.

The first step is to define bins of hydrology considering the total natural energy of each scenario. In other words, based on the capacity of hydro plants in one scenario to "convert the water of the reservoir in energy" clusters can be defined. If there's correlation among hydro plants and renewable sources, it is possible that in the same cluster, the generation of renewables is close to that of hydroelectric plants. In other words, if the hydro plants available power is large, probably it also occurs for the renewables. (in case of positive correlation).

Then, the second step consists in defining bins according to the hours of the day. For instance, if the system has solar and wind sources, but higher solar generation then wind, it's possible to observe a change in the total available power at 7a.m or 8a.m. due to the sunrise. Then, looking at the total available power in each hour, it is possible to define bins.

Thereby, for each bin (composed by hours of renewable generation, correlated with hydrologic scenarios), the IS optimization and MCMC proposed in this work could be applied for the multi-area reliability evaluation.

7.1.2 Control Variables

The results obtained from a multi-area reliability assessment can be extended to a composite reliability evaluation. In order to do so, it's proposed to consider the results of a multi-area analysis as control variable, as will be followed described.

Control variable is a method of Variance Reduction and it consists of using a new variable that is *highly* correlated to other one to reduce the variance of one desired variable. For example, consider a random variable X for which one is interested in determining its variance. Suppose that X is extremely positively correlated to Y which is a variable whose expected vale and variance are already known. Consider a new random variable Z defined as:

$$Z = X - Y + E(Y) \tag{50}$$

Where $E(Y)$ corresponds to the expected value of Y.

The expected value of Z is:

$$E(Z) = E(X) - E(Y) + E\big(E(Y)\big) \;\; or \;\; \boxed{E(Z) = E(X)} \tag{51}$$

Thereby the expected value of the variable under interest is the same of the new defined variable Z.

The variance of Z is defined as:

$$Var(Z) = Var(X) + Var(Y) + Var\left(E(Y)\right) - 2Cov(X,Y) + 2\,Cov\left(X, E(Y)\right) - 2\,Cov(Y, E(Y)) \quad (52)$$

Simplifying Equation (52):

$$Var(Z) = Var(X) + Var(Y) - 2Cov(X,Y) \quad (53)$$

From the previous equation in can be noted that if variable X and Y are extremely correlated the variance of Z can be smaller than the variance of X.

Now, if the variable $E(X)$ and $Var(X)$ corresponds respectively to the expected value and variance of the LOLP of a multi area reliability evaluation, the LOLP value for a composite reliability evaluation could be determined considering the multi-area values as control variables. This approach can be considered in cases which the LOLP expected values of multi-area reliability evaluation and composite reliability evaluation are significant correlated. This is usually true since transmission lines' failure rates are much smaller than generators failure rates (considered in the multi-area reliability evaluation). In other words, it means that most scenarios without load shedding in the multi area reliability assessment also result in no load shortage in the composite reliability evaluations.

Once a multi-area reliability assessment has been concluded, the system LOLP estimated value is known $LOLP(MA)$. Considering the methodology proposed in this thesis, the following steps can be conducted to use the results of the multi-area reliability evaluation to access the composite reliability evaluation.

1. From the "tilted" distribution of each area, draw one sample of the total available power. These samples result in a vector *i* containing the available powers in each area.
2. Determine the multi area loss of load probability $LOLP\left(MA^i\right)$.
3. From the given sample of the total available power in each area (vector *i*) is possible to determine the operating state for each generator in each area. Then, one operative state (on or off) of each circuit in the system should be sampled (by Monte Carlo Standard Simulation).

4. Once the circuits status had been sampled, the LOLP value for G&T could be determined. $\left(LOLP\left(G\&T^{\,i}\right)\right)$

For each event (operative status for generation and transmission), calculate the

$$Z^i = LOLP(G\&T^{\,i}) - LOLP(MA)^i + LOLP(MA)$$

For each Z^i calculate its expected value and variance value until reaching the relative uncertainty. Since the relative uncertainty is measured as the relation between the standard deviation and the average value, the usage of the control variable Z is useful since Z may have a smaller variance which results in a faster convergency of the composite reliability evaluation.

Control variables have already been applied in the past for calculating reliability indexes for G&T studies. OLIVEIRA *et al.* [30] proposes applying control variables to evaluate composite reliability considering generation capacity indexes (resulted from an HL1 analysis) as control variable. In [30] it is also suggested to use transmission outages as control variable for composite analysis.

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